ESTIMATING OUTPUT PARAMETERS OF MULTI-STAGE PRODUCTION PROCESSES WITH RANDOM YIELDS

By

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B.S.I.E., King Mongkut's Institute of Technology Thonburi, 1997

A Thesis
Submitted to the Faculty of the
Graduate School of the University of Louisville
Speed Scientific School
as Partial Fulfillment of the Requirements
for the Degree of

Master of Science

Department of Industrial Engineering
University of Louisville
Louisville, Kentucky

December 2001

UMI Number: 1408013

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

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A Thesis Approved on

July 20, 2001

by the Following Reading Committee:

Thesis Director

ACKNOWLEDGEMENTS

The author would like to recognize the people who helped the author to finish this thesis. Almost all of the success of the thesis has come from Dr. Gail DePuy who gave the author a lot of ideas that were very useful. Not only was she a valuable resource but she also provided good guidance.

Furthermore, Dr. Gail DePuy and Dr. John Usher also helped the author by letting the author use their software to compare the result of his thesis. The author would like to thank Dr. Ellen Brehob for serving on the thesis committee.

Also, the author would like to thank his parents, Wirat and Surape, for giving him an education and the Industrial Engineering faculty at the University of Louisville for supporting everything to complete the thesis.

ABSTRACT

This thesis describes the development of a method to estimate the output parameters from multi-stage batch production processes with random yields. There are many factors that affect the output of a production process, and therefore the associated mean, variance and expected profit of the output is often difficult to estimate.
This thesis investigates the effectiveness of several estimation methods. Estimated output parameters are compared with actual output parameters for several problem instances. Although software programs may be able to provide the exact value, it takes a lot of time to find the exact solution for large values of input flow into the production process.

The goal of this thesis is to estimate mean and variance outputs quickly for various scenarios. The results show that the best estimation method is calculated using equations based on the minimum mean and the minimum variance for random yields. The best estimation method was found to be within 1.09% from the actual mean, within 8.91% from the actual variance, and within 4.42% from the actual profit for the problem instances investigated.

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CHAPTER I

INTRODUCTION

Many billions of dollars are invested in the inventories of manufacturing companies, which causes large interest costs. A small decrease in the inventory and/or production costs can increase the profit substantially. Especially in the case of scarce capacity, efficient production schedules are fundamental for on-time delivery. To support decision makers by improving their manufacturing resource-planning system with appropriate methods is one of the most interesting challenges of production planning.

For a manufacturing company, production planning and control systems are necessary to ensure adequate supply to meet demand. Production is the process of managing resources in order to create a product and it is responsible for converting raw material into products.

This thesis investigates a specific production issue to estimate the output of a manufacturing process. These estimates can be used for planning processes. Especially, it is about the analytical methods used to support the production function. These methods could be used within a computer-based production planning and control system to aid in the estimation of the uncertain output of products.

Production Planning

Production planning is formally defined as "the process of envisioning, conceptualizating, developing, producing, testing, commercializing, sustaining, and disposing of organizational offering to satisfy consumer needs/wants and achieve organizational objectives. By this definition, product planning is certainly a broad and complex endeavor, comprising numerous issues and activities, many of which are cross-disciplinary in nature" [1]. The production planning should be concerned with estimating the output of a production process, which is necessary for a long list of other activities in an organization.

For a company to satisfy the customer's demand, the production planning method often used is material requirement planning (MRP), which is a basic tool for performing the detailed material planning function in the manufacture of component parts and their assembly into finished items. MRP is used by many companies that have invested in batch production processes such as the one that is considered in this thesis. The MRP's managerial objective is to provide the right part at the right time to meet the schedules for completed products. To do this, MRP provides formal plans for each part number, whether raw material, component, or finished good. Accomplishing these plans without excess inventory, overtime, labor, or other resources is also important.

Production costs

For the production plan, costs are always significant. There are two kinds of costs, fixed and variable cost. Fixed costs usually come from setting up the production process.

Variable costs are the costs associated with labor, material, and overhead etc, with each piece produced.

Most production planning decisions affect inventory levels, raw material, work in process or finished goods inventories. The inventory costs that are significant are holding cost, procurement cost, and back order or lost sale costs. These costs will be further described in Chapter III.

Predicting Output

A significant tool of the production planning process is predicting the output of the batch process to be compared to the consumer's demand. Unfortunately accurate predictions are difficult. Predicted output is an important input to most other tools of production planning and scheduling models. The results of other methods depend on the accuracy of the prediction. Although the software program can calculate the exact outputs, it takes a lot of time to solve the problems that have large numbers of input.

For example, software developed at the University of Louisville by DePuy and Usher can be used to calculate the exact output distribution for any multi-stage production process with binomial yields. However, the run times for this software become quite lengthy as the production runs become large. To demonstrate this concept, a 14 stage production structure was developed and the exact output distribution was calculated for various production levels. The software was run on a Pentium III PC.

Production Level	Run Time
200 units	13.5 seconds
1,000 units	350 seconds

2,000 units	1,404 seconds
10,000 units	35,100 seconds

Good simple methods of estimating the output mean and variance of multi-stage batch process with random yields are not readily available. This thesis attempts to develop such a method for estimating the output parameters in short time. The company that can develop an estimation method that gives the output of products accurate to customer's demand will be successful in the business.

Variable Production Yield

In most factories, there are many different process operations that involve assemblies with serial and parallel structure. The products that already pass the qualification are shifted to the consumer, but the products that fail the qualification are rejected. Basically, it is impossible for the operation stage in each process to produce the 100 % accepted yield without rejects. The number of rejects can be large or small depending on the variable factors in each stage of operation such as machine, material, method, operator, and so on. Therefore the yield in each operation greatly affects the mean and variance of final outputs.

Engineers who work for a long time in the process will understand the behavior of the process system well. They can estimate the parameter outputs more accurately than people who do not know much about that process. Therefore, it is very difficult to estimate the mean and variance of the process for new people because the process structure has many stage operations. It is also complicated for new engineers who have to be concerned with the variable factors that are involved in the output of the product (mean and variance). In this thesis, 8 multi-stage structures will be created in different patterns to investigate the method for estimating the output parameters.

Problem Description

The Process Structure

In the manufacturing, there are two fundamental structures [2], serial process and parallel process.

The Serial Process

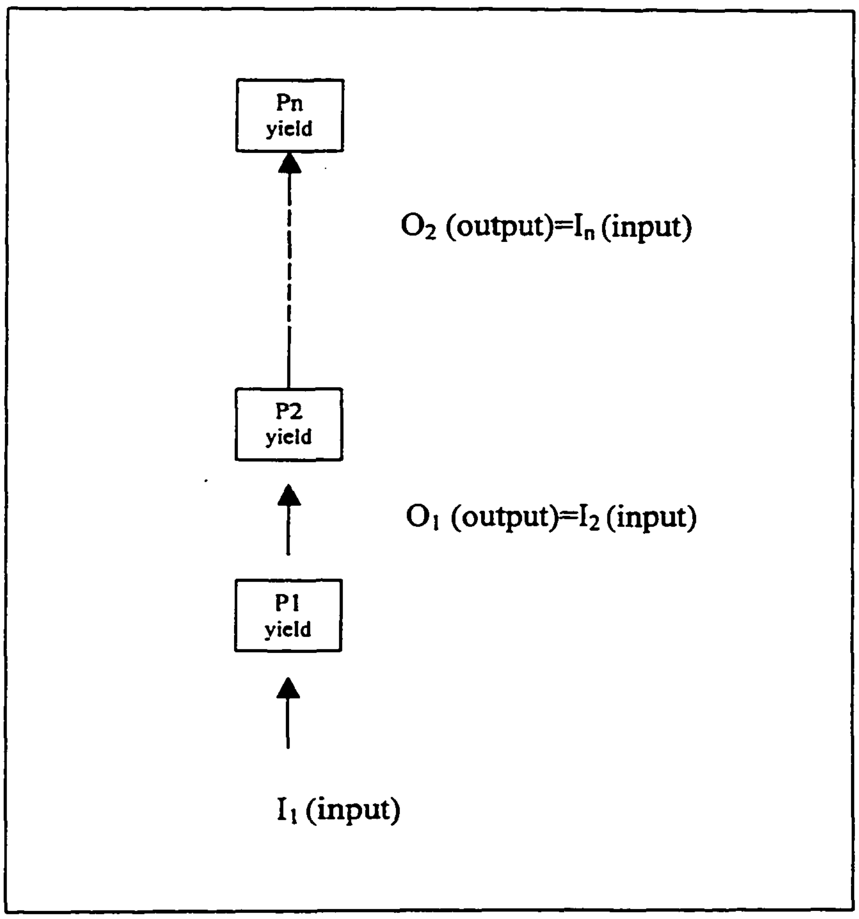

Figure 1 - Operations in Series

For purposes of discussion, assume that n serial operations are numbered sequentially from 1 to n, where operation n represents the last operation in the sequence. I_n represents the input to operation n, and P_n represents the yield in operation n as follows. Let O_n represent the output from operation n. O_n is calculated.

$$O_n = I_n P_n \tag{1}$$

Starting with operation 1, I_1 represents the input to the first operation in the series. Operation 1 has a random yield from uniform distribution that is represented by P_1, and O_1 represents the output from the operation 1 which becomes the input to operation 2. This process continues until the final product is output from operation n.

The Parallel Process

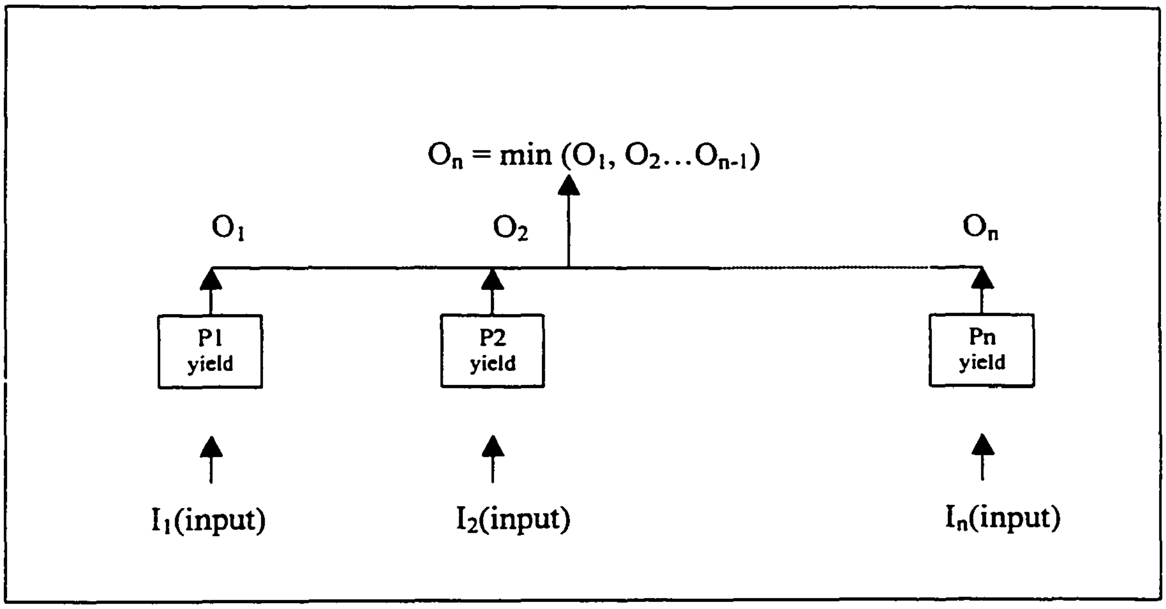

Figure 2 - Operations in Parallel

In the real process, multiple components may be assembled or joined in some way to form a subassembly of the final product. Figure 2 illustrates the parallel nature of n operation being formed into a subassembly. Since the quantity of subassemblies produced cannot be greater than the minimum of the individual component quantities, note that:

$$O_n = \min (O_1, O_2 \ldots O_n) \qquad (2$$

Starting with operation 1, 2... n, I_n represents the input for operations 1,2... n respectively. The outputs of product in each operation depend on the yield that is represented by P_1, P_2..., P_n. The final output will come from the smallest quantity of O_1, $O_2 \ldots O_n$.

The Combination Process Structure

For a combination of serial and parallel operations, the equations (1) and (2) can be applied sequentially to determine the inputs and outputs of each operation. Figure 3 shows a product structure with combinations of serial and parallel operations.

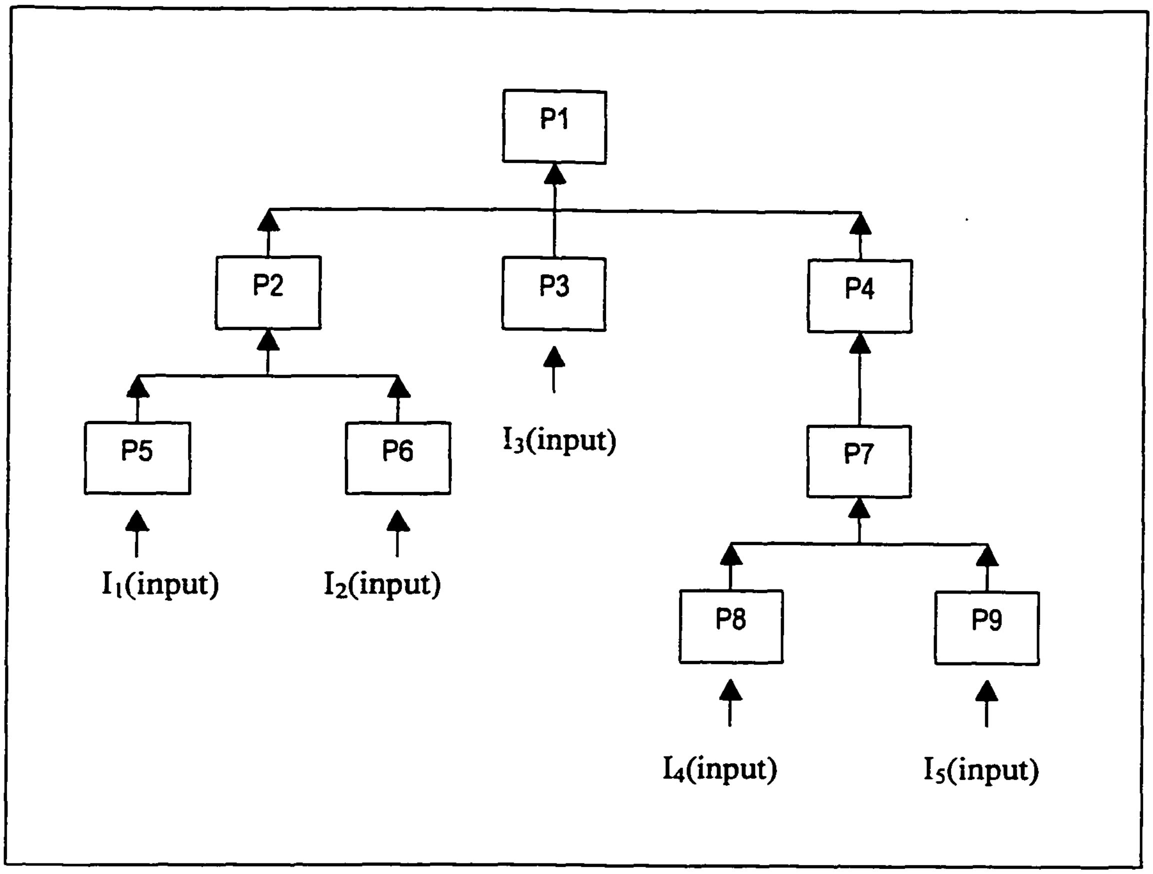

Figure 3 - Combinations of Serial and Parallel

The material inputs are transferred to stage outputs using P5, P6, P3, P8, and P9. The minimum output from stages 8 and 9 is the input for stage 7 and then the materials flow to stage 4. Again the minimum output from stages 5 and 6 are sent to the stage 2. Stage 3 has initial input. After stages 2, 3, and 4 finish operation, they will return the outputs. The smallest output is sent to stage 1 to produce the finished goods of this process structure.

The output of a process depends on the product structure (i.e. the combination of series and parallel operations), yields, and raw material inputs. In reality, each production yield is a random variable. For a given product structure and set of raw material inputs different outputs could be realized in successive batches due to random yields. Therefore the process output is also a random variable.

Estimating the parameters of these batch process outputs is important to production planning. This thesis develops a simple method to estimate the mean and variance of the output of any general product structure with random yields.

CHAPTER II

LITERATURE REVIEW

Currently, manufacturing processes can be very complex. It is difficult to develop methods to maximize the total profit because there are many factors that affect the cost, and therefore the companies will require better training and preparation for competition than ever before. As powerful computing capability is brought directly to the factory, sophisticated mathematical models will be used more than ever.

In the study of multiple lot- sizing problems with rigid demand, Sy-Ming Guu [3] provides a good description of the affects of general cost structures with the interrupted geometric yield such as the binomial and discrete uniform. The cost structure and yield distribution are two main factors to determine the behavior of such problems. Guu [3] presents results characterizing the behavior of the optimal total cost function and optimal lot sizes.

For recent review of lot sizing models with random yields, readers are referred to Yano and Lee [4]. They provide a discussion on the modeling of cost affected by random yields, approaches for modeling yield uncertainty, and all measures of performance. They review discrete and continuous time models, single and multi stages of production process, and single and multiple periods. In particular, the optimal (initial) run sizes with the binomial yield and the discrete uniform yield have been studied by Beja [5] and Anily [6], respectively.

Then Beja [5] and Anily [6] provide efficient computational procedures for finding optimal run sizes for both yield distributions. In addition to Beja [5], Grosfeld-Nir and Gerchak [7] develop the structural property for binomial and uniform yields but they do not extend to a general yield distribution.

Yano and Chan [8] have studied a two-component case in which procurement decisions are made for several assembly components with mutually independent yield rates. Gerchak, Wang and Yano [9] also consider the case of multiple components with identical cost characteristics and yield rates. They derive optimality conditions for a variety of situations considering different disposal costs, and salvage values. Lee [10] considers a serial production line consisting of both production and assembly station under the assumption that yields are stochastically proportional to input quantity. He shows that the optimal input policy is formed by a set of critical numbers, which turn out to be easily computed when the production system consists of all assembly operations.

DePuy and Usher [11] provide approaches for determining starting batch sizes for a combination of serial and parallel manufacturing operations using exact binomial calculations at each stage. They develop a fast computer heuristic for solving this problem when the yield at each operation is assumed to be a binomial random variable. When the customer order quantity is large, the evaluation of the binomial yield distribution becomes cumbersome. Therefore a normal approximation to the binomial yield and a standardized loss function is used in this heuristic. The approach searches for the starting batch size that maximizes an expected profit function composed of the cost to process each unit in operation.

Furthermore, Sherman [12] provides a batching technique for estimating the variance of point estimators that are computed by simulation experiments. The sample mean is the most common statistic used to estimate the steady state mean of a simulation. In the literature he shows that the sample median has some attractive properties beyond those it has in the independent data setting. Specifically, he shows that the sample median loses very little in efficiency for positively correlated output, and the batch variance estimator of the sample median has comparable bias to that of the sample mean in estimating its asymptotic variance.

While the multiple production runs are studied implicitly without inspection cost, Abraham Grosfeld-Nir, Yigal Gerchak, and Qi-Ming He [13] provide these costs as a key part of the problem. They found the optimal production lot size depends on the inspection cost. They provide a framework to calculate the optimal batch and expected number of inspections for any yield pattern, as well as for any inspection procedure to fulfill the contract requirement and inspection.

To develop material requirements planning methods, Mersch [14] provides a methodology for material requirements planning of multiple level, multistage assembly system where the production yields at each stage are random to create a model of yield and develop methods of yield loss compensation in the production planning function. His MRP records are created for fundamental product structures and lot sizing decisions are simulated based on the demand, relevant costs, and randomized yield rate data.

To gain the information on production variable yield and production cost in multi-stage structure, this thesis estimates the output parameter of products and expected profit cost in different ways. It shows that the multiple stage process structures have been created to simulate the output parameters (mean and variance) of products by generating random yield in each operation and material input to find the best method for estimating the output parameters of products and the expected profit.

CHAPTER III

METHODOLOGY

This chapter presents a methodology to predict the output parameters and expected profit from multi-stage structures with random yields. In manufacturing, products are produced by different sequential structures of processes with in different yields. This experiment will generate a random number of initial inputs and yields at each stage, and also create different structures to represent the real process in manufacturing.

General Methodology

1) Create 8 multi-stage structures to simulate the experiment.
2) Generate material inputs for the first stage and production yields in each stage by using uniform distributions.
3) Estimate mean and variance of output for each structure using a variety of estimation techniques.
4) Compare mean and variance outputs from estimation to the exact solutions from software program.
5) Calculate the expected profit using the same parameters.
6) Compare expected profit with the exact solution.
7) Evaluate the methods to find the best estimation method.

Generation of Input Parameters using Uniform Distributions

There are only two possible outputs of the operation stage in manufacturing, such as success and failure. Additionally, they are assumed to be independent in the output, i.e. each output does not affect the other output.

It is assumed that the probabilities of success and failure are p and q, respectively ($p + q = 1$). For example, suppose that products are inspected in each stage for which the probability of acceptance is p while the probability of rejection is q. Then the product passes the stage, which is the probability of accepted product.

The random number is the number of success, which can range from 0 to 1, and the distribution is a function of two parameters, success or failure.

In the experiment, material inputs and yields are randomly generated for 8 structures by using a uniform distribution for each stage operation. They will be simulated by using the Monte Carlo method. The range of material inputs is from 65 to 250, and the range of the yields is from 0.65 to 0.99.

Monte Carlo Simulation Method

The Monte Carlo simulation method [15] is a powerful method that can be a valuable aid in simulating the output of the production process. It is a good method that yields solutions to complex multi-dimensional problems. This thesis uses the Monte Carlo simulation to calculate mean and variance output of the experiment for multi-stage structure.

First, initial raw material inputs are generated in the first level of the structure by using a uniform distribution (65,250). Then these materials are put into the process structure. They flow to the final stage of the structure to return the output of products.

In the same time, the process structure has different yields to simulate the yield in each of multi-stage structure process by a using uniform distribution (0.65,0.99).

For a given product structure, many simulations are run using randomly generated input values and yields to calculate the output parameters of the product. The simulation is run many times for each product structure, and the mean and variance of the output distributions are calculated by using the Monte Carlo simulation. These output parameters will be used to evaluate the appropriateness of the estimation methods.

Estimation of the Output Parameters

When the experiment generates random inputs and yields in each structure and flows the material input to the process line, there are many ways to estimate the output parameters (mean and variance) of product structure. This section will show the estimation methods investigated in this thesis.

To estimate the output parameters, a multistage structure is separated into "paths". At first, the multiple stages are separated into paths by following the initial input line to the final stage. In each path, the values of initial input are calculated by using a serial formula that is provided by DePuy and Usher [11]. They assume each process stage is independent, the probability p' that an individual unit successfully completes all N stages is simply:

$$p' = \prod_{i=1}^{N} p_i \tag{3}$$

After calculating the output values in each path, there are two different methods to calculate the variance parameter, Method A and Method B, and five alternative options in each method are used to estimate the output parameters.

There are five options and two different calculation methods of variance to compare with each other. The one that gives the closest value to the actual value that will be the best estimation method.

Options for Estimating Output Parameters

Option 1 = Select minimum mean and variance of final process

Option 2 = Select minimum (mean-3σ) and variance of final process

Option 3 = Select minimum mean and maximum variance of final process

Option 4 = Select minimum mean and minimum variance of final process

Option 5 = Select minimum mean and average variance of final process

Formulas for Calculating the Parameters

The estimation methods developed in this thesis separate the multiple stages into paths to calculate the output parameters (means and variance). There two methods, Method A and Method B, that use the serial formula to calculate the output parameters in each path using the following the formulas.

Method A

N = number of stage

p = yield in each stage

I = initial input in each stage

Calculation for mean output parameter in each path

$$p' = \prod_{i=1}^{N} p_i$$

$$\text{Mean A} = \mu_A = I * p' \quad (4)$$

To calculate variance output of method A, the formula is based on calculations to determine the variance of a nonlinear function of random variables. For example, Montgomery [16] shows the mean and variance of a nonlinear combination of random variable is $\mu_y = g(\mu_1, \mu_2, \ldots \mu_n)$ and $\sigma_y^2 = (\mu_2^2\mu_3^2\mu_4^2\,\sigma_1^2) + (\mu_1^2\mu_3^2\mu_4^2\,\sigma_2^2) + (\mu_1^2\mu_2^2\mu_4^2\,\sigma_3^2) + (\mu_1^2\mu_2^2\mu_3^2\,\sigma_4^2)$. They will be developed to equation (5) to calculate the variance values for method A.

Calculation of variance output parameter in each path

$$\sigma_A^2 = I*((p_1*(1-p_1)*(p_2*p_3*p_4\ldots p_n)^2) + (p_2*(1-p_2)*(p_1*p_3*p_4\ldots p_n)^2)+(p_3*(1-p_3)*(p_1*p_2*p_4\ldots p_n)+\ldots\ldots\ldots(p_n*(1-p_n)*(p_1*p_2*p_3*\ldots p_{n-1})^2)) \quad (5)$$

Method B

Calculation for mean output parameter is same as Method A that is shown in equation (4).

Calculation of variance output parameter in each path

$$\sigma_B^2 = I * p' * (1-p') \quad (6)$$

This equation is based on equations from DePuy and Usher [11].

The Expected Profit Function

Every company needs to decrease costs so that profit is maximized. This research includes an expected profit equation to evaluate solutions in terms of costs and revenues. This expected profit equation will be used as another method of evaluating the output parameter estimates developed in this thesis.

The profit function includes sales revenue, variable costs of processing at each stage, per unit shortage cost and per unit overage cost.

Costs

These costs involve the cost of producing one unit of output during regular working hours. Included in this category are the actual payroll costs of regular employees working on regular time, the direct and indirect costs of material, and other manufacturing expenses.

In some cases it may be necessary to incur a shortage that is represented by a negative level of inventory. Shortages can occur when forecasted demand exceeds the capacity of the production facility or when demands are higher than anticipated. A shortage cost is often associated with loss of customer goodwill.

Holding costs are the costs that accrue as a result of having capital tied up in inventory. If the firm can decrease its inventory, the money saved could be invested elsewhere with a return that will vary with the industry and with the specific company.

Expected Profit Equation

The estimated expected profit equation as defined by DePuy and Usher [11] is shown in equation (7), where:

N = total number of operations

T = target amount of finished product to produce

c_i = variable cost for operation i ($/unit)

s = shortage cost charged for producing less than T ($/unit short)

v = overage cost charged for each unit produced in excess of T ($/unit over)

G = sale price of finished product ($/unit)

$$\text{E(Profit(}I_1\text{))} = (G+s)\left(\mu - \sigma\ L(z)\right) - \sum_{i=1}^{N} c_i \bar{x}_{I_i} - sT - v\ \sigma\ L(z) \qquad (7)$$

Derivation of this equation is shown in equation (8) through (17).

The estimated expected profit equation (7) will be compared with the exact expected profit value from equation (9) that is included in the DePuy and Usher [11] software.

$$\text{E(Profit(}I_1\text{))} = \text{E(revenue)} - \text{E(variable cost)} - \text{E(shortage cost)} - \text{E(overage cost)} \qquad (8)$$

$$\text{E(Profit(}I_1\text{))} = \left[\sum_{x \le T} GxP(O_N = x) + \sum_{x > T} GTP(O_N = x)\right] - \sum_{i=1}^{N} \sum_{x} c_i xP(I_i = x)$$
$$- \sum_{x<T} s(T-x)P(O_N = x) - \sum_{x \ge T} v(x-T)P(O_N = x) \qquad (9)$$

These binomial probabilities will be used in equation (9).

I_i represents the material input to operation i.

O_i represents the output from operation i.

N represents the operation stage in series from 1 to N.

$$X_i = \begin{cases} 1 \text{ if the product passes the qualification to the next operation} \\ 0 \text{ if the product fails the qualification.} \end{cases}$$

$$P(O_i = x) = \sum_{j=x}^{I_1} P(O_i = x | O_{i-1} = j) \cdot P(O_{i-1} = j)$$

$$= \sum_{j=x}^{I_i} C_x^j (1-p_i)^x p_i^{j-x} \cdot P(O_{i-1} = j) \quad (10)$$

The expected profit in equation (10) is for the binomial probabilities. It is not appropriate to calculate expected profit for large T. To simplify, DePuy and Usher [11] utilize the normal approximation to the binomial distribution, which are presented in equation (11).

$$\text{E(Profit(} I_1 \text{))} = \int_0^T G\, x\, f_{O_N}(x)dx + \int_T^\infty G\, T\, f_{O_N}(x)dx - \sum_{i=1}^{N} c_i \int_0^\infty x\, f_{I_i}(x)dx$$

$$- \int_0^T s\,(T-x)\, f_{O_N}(x)dx - \int_T^\infty v\,(x-T)\, f_{O_N}(x)dx \quad (11$$

Where $f_i(x)$ represents the probability density function of the normal distribution for the i^{th} operation.

Additionally, equation (11) can be simplified to the standardized loss function, L*(z)*, Nahmias [17] uses the standardized loss function $L(z) = \int_z^\infty (x-z) f(x)dx$ to develop the equation (12) following below.

$$\int_T^\infty (x-T) f(x)dx = \sigma\, L\left(\frac{T-\mu}{\sigma}\right) = \sigma\, L(z) \quad (12$$

After that equation (11) is developed from equation (12). The values of standardized loss function, *L(z),* are available in Appendix A.

$$\int_T^{\infty} x\ f(x)dx = \int_T^{\infty} (x-T)\ f(x)dx + \int_T^{\infty} T\ f(x)dx = \sigma\ L(z) + T(1-F(T)) \qquad (13$$

and

$$\int_0^T x\ f(x)dx = \int_0^{\infty} x\ f(x)dx - \int_T^{\infty} x\ f(x)dx = \mu - (\sigma\ L(z) + T(1-F(T))) \qquad (14$$

Equation (15) is created by replacing equation (12), (13), and (14) into equation (11). It is shown below.

$$\text{E(Profit(} I_1 \text{))} = G\left[\mu_{O_N} - \left(\sigma_{O_N}\ L(z) + T(1-F(T))\right)\right] + GT(1-F(T)) - \sum_{i=1}^{N} c_i \bar{x}_{I_i}$$

$$- sT(F(T)) + s\left[\mu_{O_N} - \left(\sigma_{O_N}\ L(z) + T(1-F(T))\right)\right] - v\sigma_{O_N} L(z) \qquad (15)$$

To simplify, equation (15) will be changing to equation (7). It is the equation for the normal approximation to serial processes with binomial yield.

$\bar{x}_{I_1} = I_1$ represents the average input into operation i

F(T) is the cumulative normal distribution of the final operation output

$$\bar{x}_{I_i} = T \prod_{j=1}^{i-1} (p_j) \quad \forall i = 2,\dots,N \qquad (16)$$

$$z = \frac{T - \mu_{O_N}}{\sigma_{O_N}} \quad (17)$$

$$\text{E(Profit(}I_1\text{))} = (G+s)\left(\mu_{O_N} - \sigma_{O_N} L(z)\right) - \sum_{i=1}^{N} c_i \bar{x}_{I_i} - sT - v\,\sigma_{O_N} L(z) \quad (7)$$

DePuy and Usher [11] show equation (7) that provides an accurate estimation of the expected profit equation, which was already presented. Equation (7) will be used to estimate the expected profit by simulation. It will be compared with the expected profit value used in the software program in equation (9).

The Process Structure Model

The experiment creates eight different structures to simulate the output parameters of each structure. In each process structure, there are five alternative options to estimate the output parameter. All five alternative options are compared to the actual value that is given by the software. The appropriate method will give the option that has the output value that is closest to this software program. That option must be the best method for estimating the output parameter of the process structure. Furthermore this thesis shows how to calculate the expected profit cost by using the output parameter from five alternative options. When the value of expected profit costs are presented, they also are compared to the value to the actual value from the software program. The best method will have the expected profit cost that is closest to the software program.

As an example of the process structure see Figure 4. In each stage operation, there are different yields. They depend on the quality of process operation that has the defect of product. For each stage, the yields are generated by using a uniform distribution from (0.65,0.99).

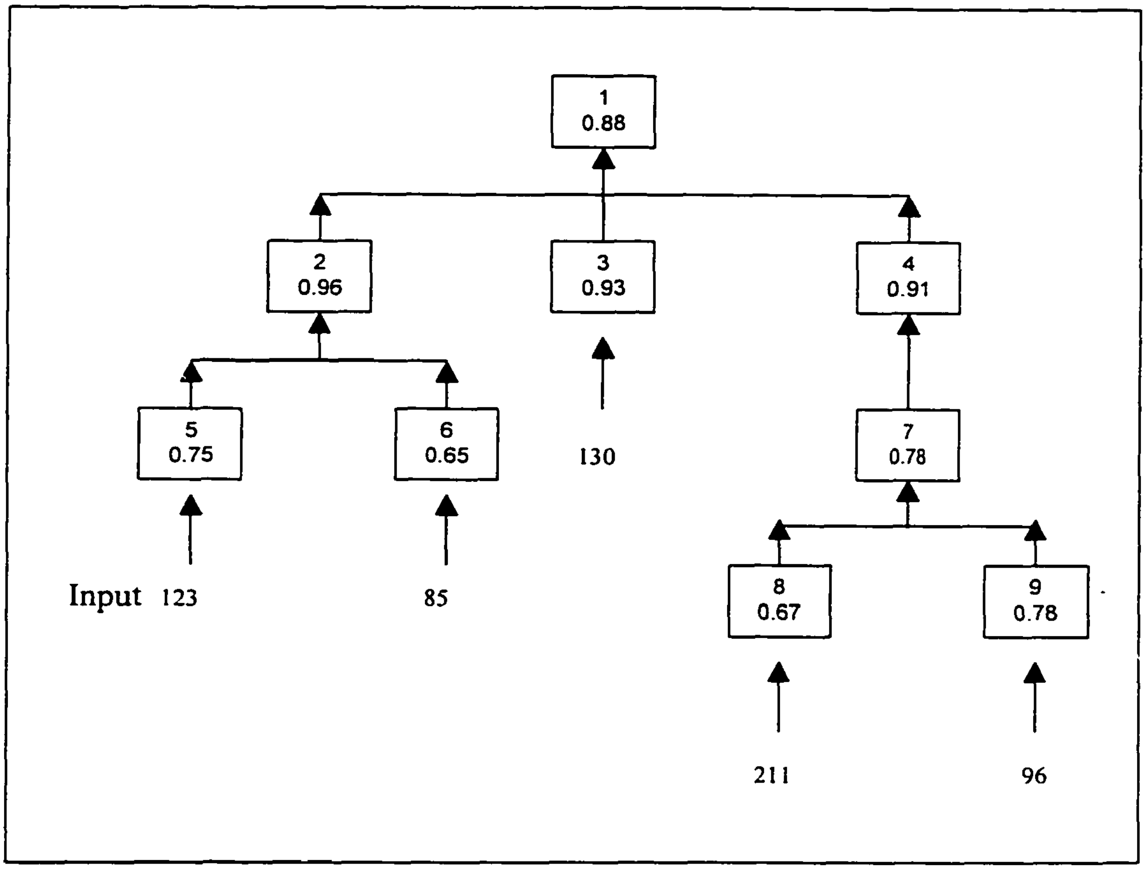

Figure 4- Process Structure 1

In Figure 4, the process structure has nine stages. It is assumed the final stage of this structure is stage number 1. Figure 4 shows sample values for material inputs and yields. To calculate the output parameters, it will be separated into 4 levels and 5 paths as outlined in Table 1.

TABLE 1

Level Number

Level	Operation Stage
1	8 and 9
2	5,6, and 7
3	2,3, and4
4	1

It is assumed this process structure has 5 paths. The multi-stages are separated into paths that follow the initial input. Table 2 shows the first stage of each path.

TABLE 2

Path Number

Path	Initial Input Stage	Stage Numbers included in Path
1	5	5,2,and 1
2	6	6,2.and 1
3	3	3 and 1
4	8	8,7,4,and 1
5	9	9,7,4,and 1

The Process Method

Numbers of material inputs are generated by using uniform distribution (65,250). These material inputs are sent into the initial input of each process part. Figure 5 shows the process for path 1.

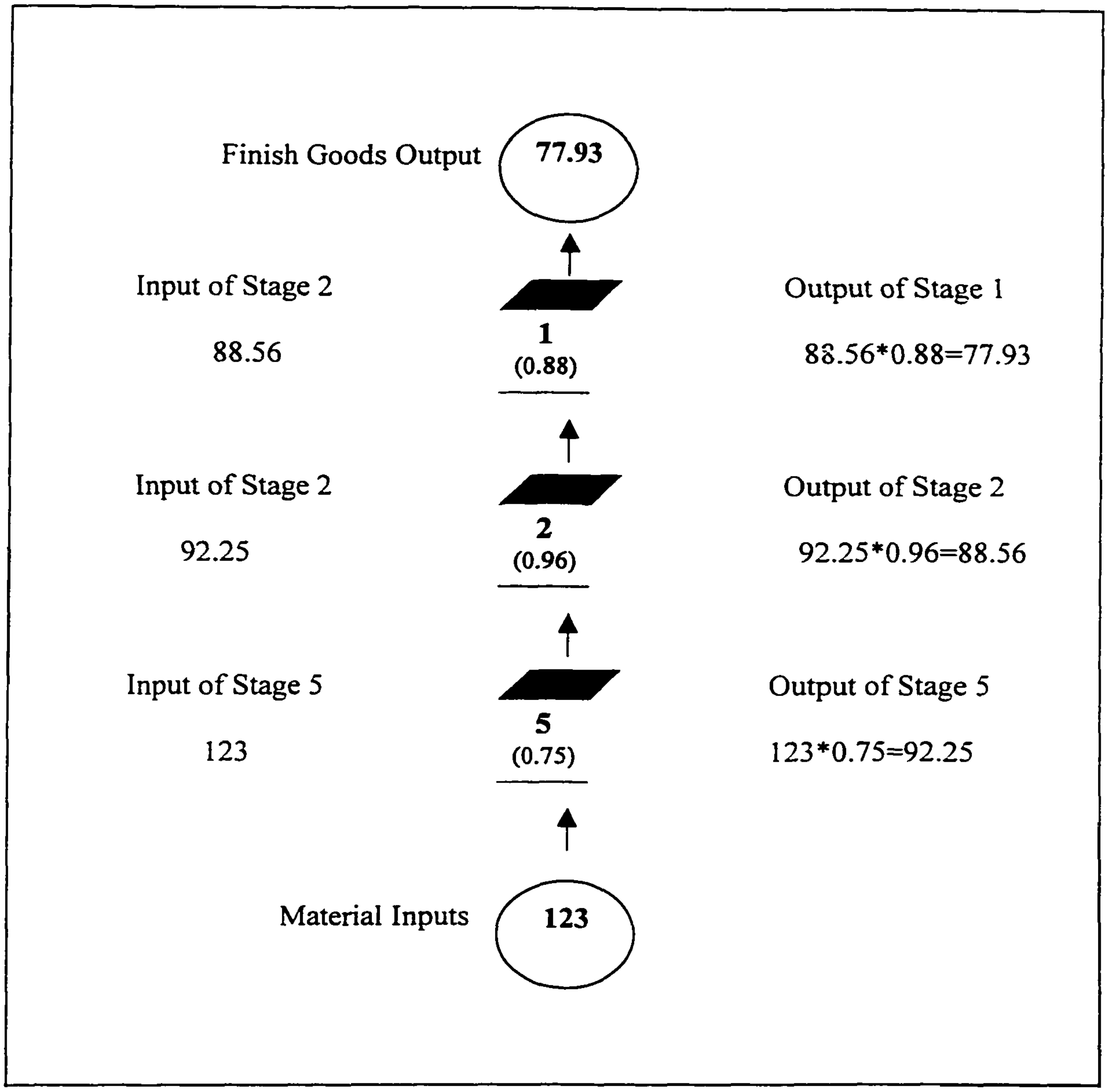

Figure 5 - Path 1 for Process Structure 1

Figures 6 shows path 2 that is done in the same way as the path1, but it uses different initial inputs and yields.

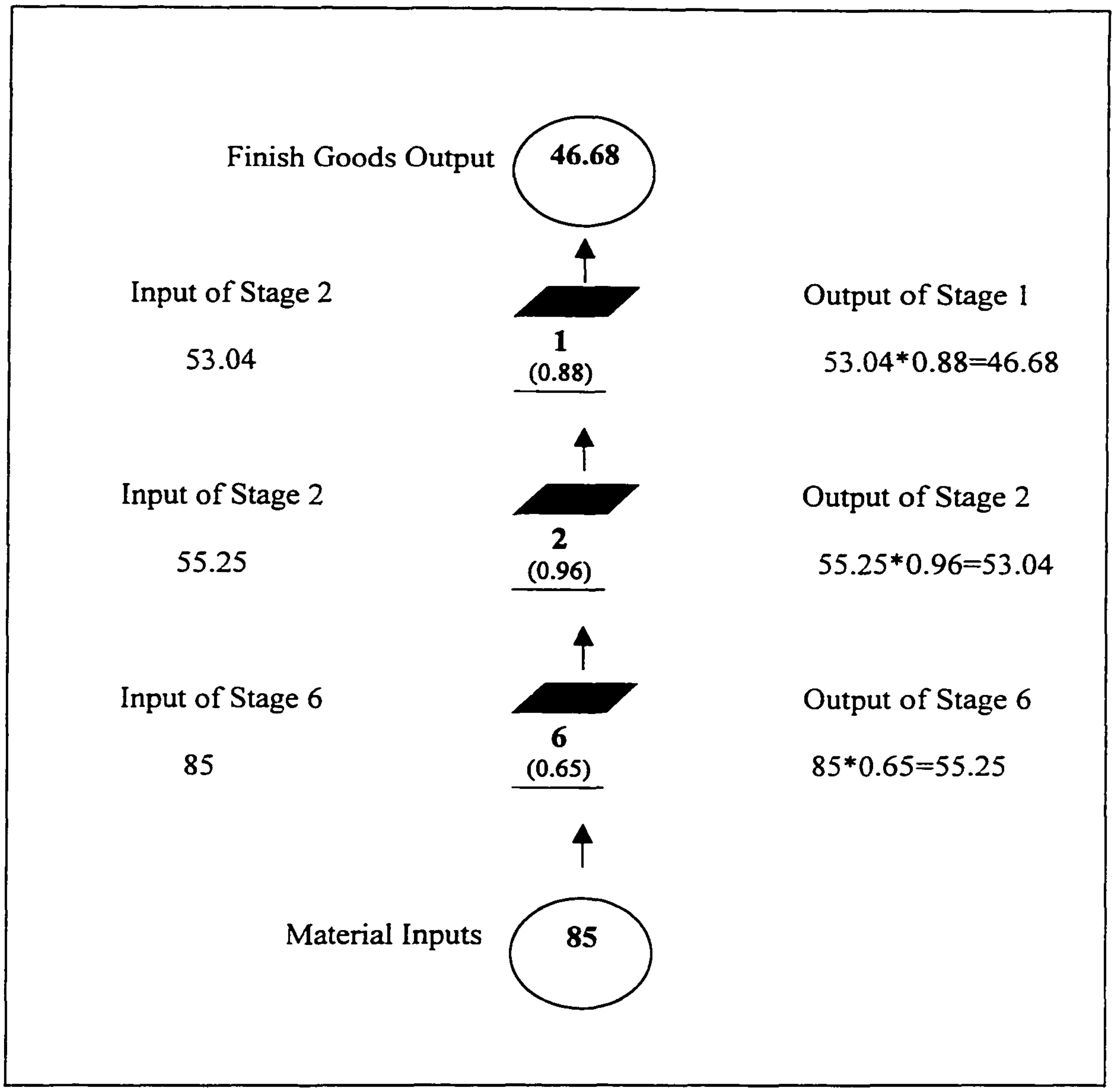

Figure 6 - Path 2 for Process Structure 1

Figure 7. shows the process of path 3.

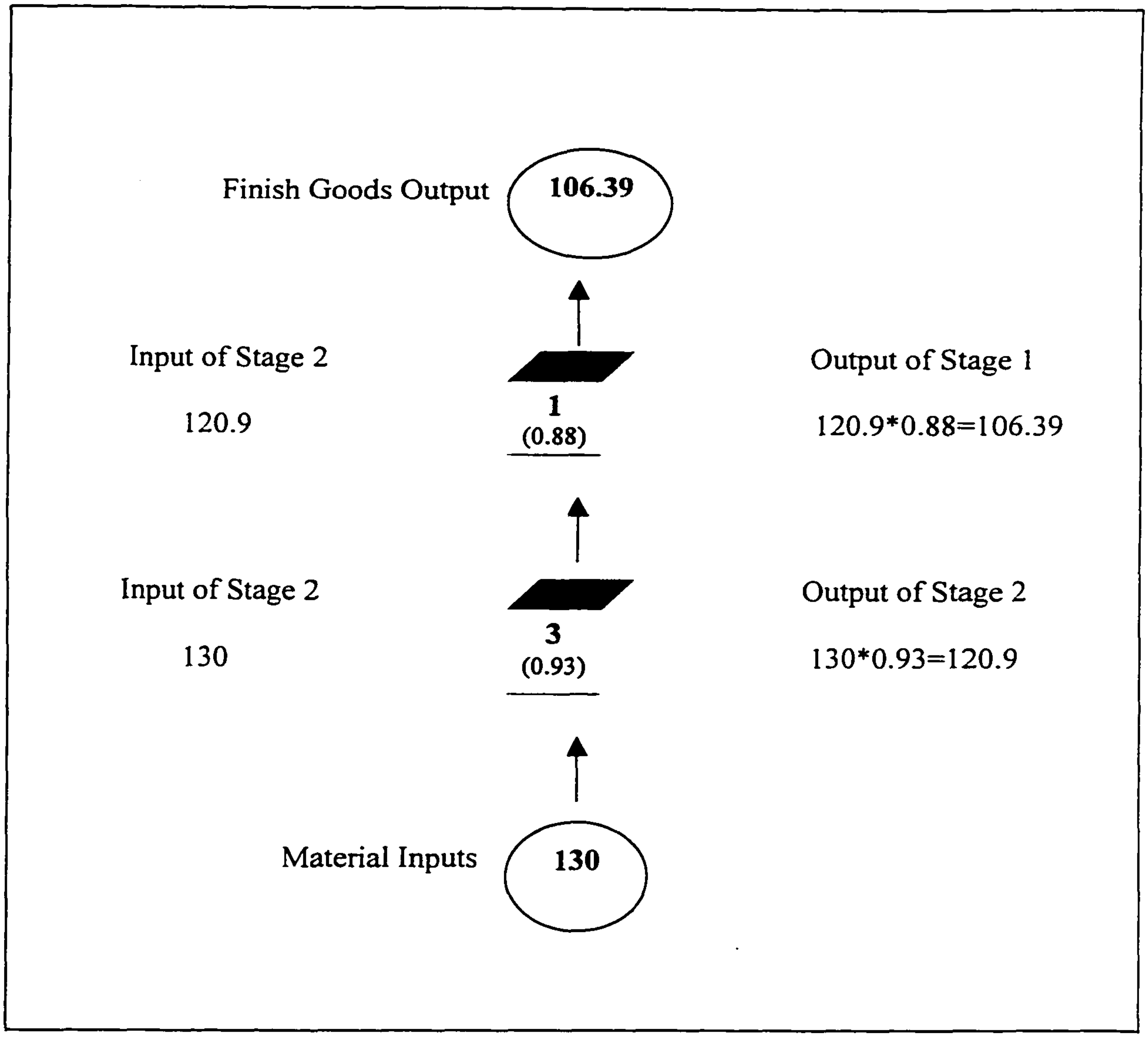

Figure 7 - Path 3 for Process Structure 1

Figure 8 shows the process of path 4.

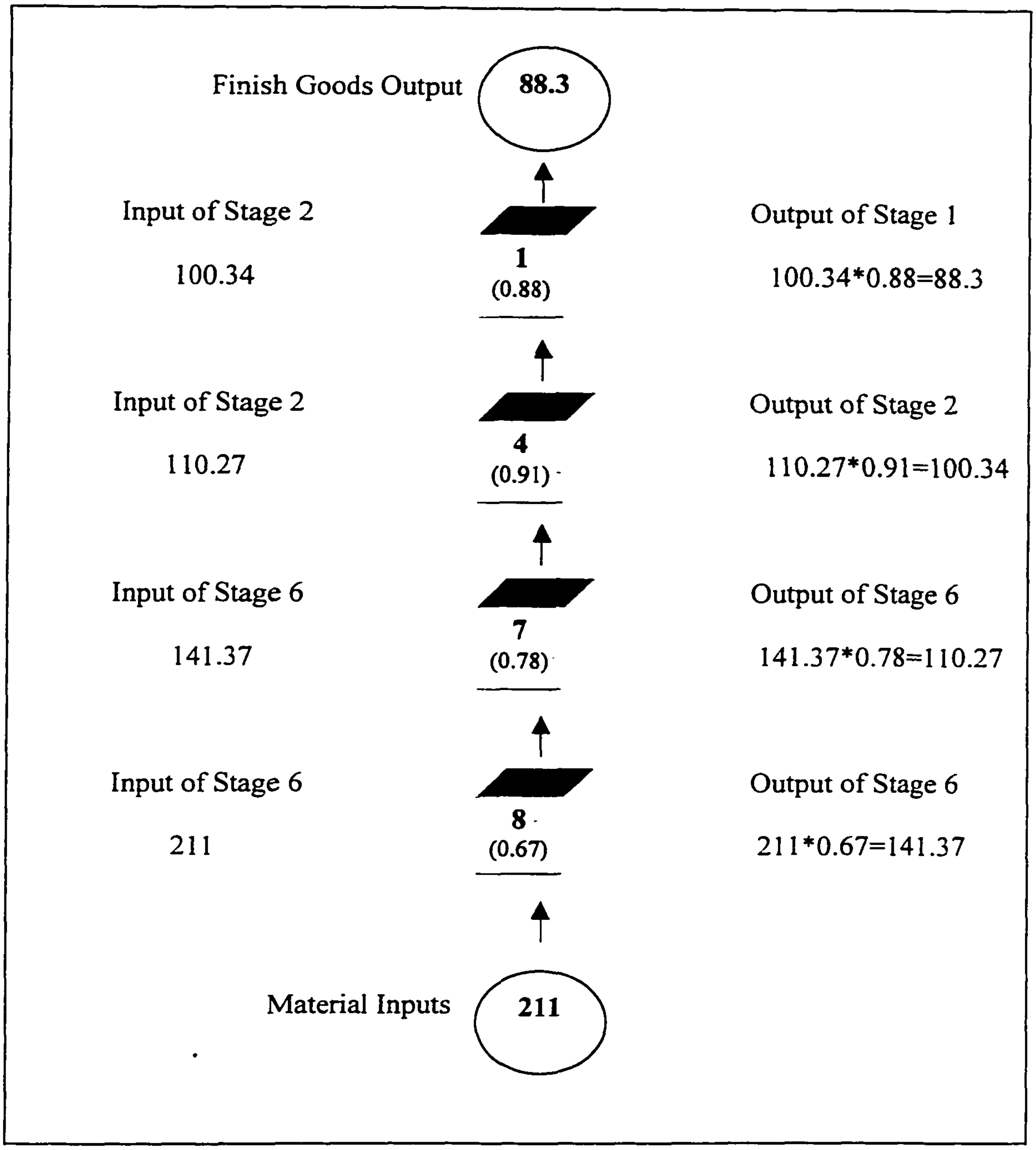

Figure 8 – Path 4 for Process Structure 1

Path 5 has four levels in the paths. It is also done in the same way as the path, but it uses different initial inputs in the first stage.

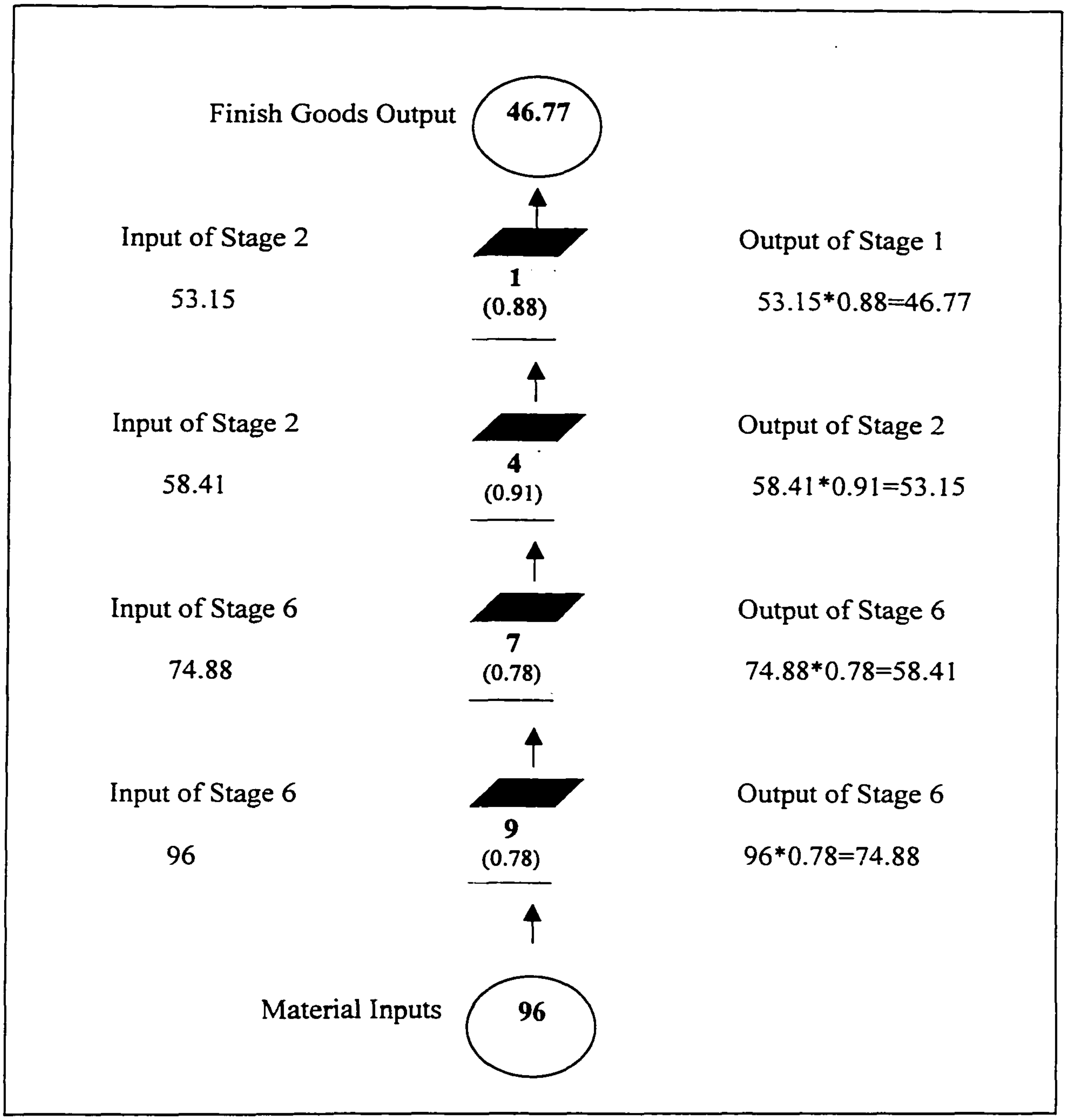

Figure 9 - Path 5 for Process Structure 1

Calculation the Mean and Variance

This is an example for path 1 (See Figure 4). Method A and B have the same equation to calculate mean. The serial equation will be used, which is referred to equation (4).

Mean of part 1 = $I_1 (p_1 * p_2 * p_3)$

= 123*(0.75*0.96*0.88)

= 77.93

This is calculation of variance method A. It is referred to equation (5)

Variance A of part 1 = $I*((p_1*(1-p_1)*(p_2*p_3*p_4...p_n)^2) + (p_2*(1-p_2)*(p_1*p_3*p_4...p_n)^2)+(p_3*(1-p_3)*(p_1*p_2*p_4...p_n)+.........(pn*(1-p_n)*(p_1*p_2*p_3*...p_{n-1})^2))$

= $123*(0.75*(1-0.75)*(0.96*0.88)^2)+(0.96*(1-0.96)*(0.75*0.88)^2)+(0.88*1-0.88)*(0.75*0.96)^2))$

= 25.25

This is calculation of variance method B. It is referred to equations (6)

$p' = p_1 \cdot p_2 \cdot p_3$

= 0.75*0.96*0.88

= 0.6336

Variance B of part 1 = $p' * I * (1- p')$

= 0.6336 *123 * (1-0.63)

= 28.55

For parts 2, 3, 4, and 5, they will be calculated the same way as path 1. It will give the results (See table 3).

TABLE 3

The Output Parameters in Path

Path No.	Mean	Variance A	Variance B
1	77.93	25.25	28.55
2	46.68	18.36	21.04
3	106.39	18.43	19.32
4	88.3	37.32	51.35
5	46.77	18.22	23.98

Mean and variance in each path will be estimated using each of the 5 options discussed on page 12.

TABLE 4

The Parameters in Option

Option	Mean A	Variance A	Mean B	Variance B
1	46.68	18.36	46.68	21.04
2	46.68	18.36	46.77	23.98
3	46.68	37.32	46.68	51.35
4	46.68	18.22	46.68	19.32
5	46.68	23.52	46.68	28.85

After the results are known, they will be compared to the exact output parameters calculated using the software program. The same input will be used in the software program. The exact output parameters are shown below

Mean = 44.41

Variance = 17.03

For this example, option 4 using variance A gives the best estimation. Many replications can be run by randomly generating different yields and initial inputs. After the simulation is run many times, the results will be evaluated. To measure the estimation method, equations (18) and (19) present the calculation of different percentage mean and variance between values from simulation and values from the software program. See the result in Table 5.

Measurement of Average Percentage Different from Exact value (APDE)

To compare the value of mean, variance, and expected profit to the exact values, the average percentage of different from the exact value is calculated following equations (18), (19), and (20).

Average Percentage of Different from Exact value = APDE

M = estimated mean

V = estimated variance

Ev = exact value

APDE (M) = [Avg. | (M- Ev. M)|/ (Ev. M)]*100 (18)

APDE (V) = [Avg. | (V- Ev. V)|/ (Ev. V)]*100 (19)

TABLE 5

The Average Percent Difference from Exact Value

Structure 1			
Method	**Option**	**Avg. Percent Different from Exact**	
		Mean	**Variance**
A	1	1.53	21.14
	2	1.53	21.14
	3	1.53	68.31
	4	1.53	21.95
	5	1.53	17.84
B	1	1.53	10.18
	2	1.64	13.45
	3	1.53	119.46
	4	1.53	9.21
	5	1.53	37.43

Calculation the Expected Profit

The equation (7) will be used to calculate the expected profit from 5 options. This is an example for method A option 1.

T = target amount of finished product to produce = 40

c_i = variable cost for operation i ($/unit) = 2

s = shortage cost charged for producing less than T ($/unit short) = 30

v = overage cost charged for each unit produced in excess of T ($/unit over) = 30

G = sale price of finished product ($/unit) = 100

μ = 46.68 from method A option 1

σ_N = 4.29 from method A option 1

$$z = \frac{40 - 46.68}{4.29}$$

$$= -1.56$$

L(- 1.56) = 1.5855

$$\text{E(Profit(}I_1\text{))} = (G+s)\left(\mu_{O_N} - \sigma_{O_N} L(z)\right) - \sum_{i=1}^{N} c_i \bar{x}_{I_i} - sT - v\,\sigma_{O_N} L(z)$$

$$= (100+30)(46.68-4.29(1.5855))-1773.15-30*40-30*4.29*1.5855$$

$$= 2007.53$$

For the other method options, they will be calculated in the same way following the mean and variance outputs of each option in many replications. This table presents the results of expected profit values.

TABLE 6

The Results of Expected (Profit)

Option	Expected (Profit) Variance A	Expected (Profit) Variance B
1	2007.53	1999.43
2	2007.53	1988.63
3	1952.40	1910.26
4	2005.5	2005.94
5	1994.19	1976.88

The same inputs and yields will be used to calculate the values by using software program. That is calculated from equation (9). It returns the results below.

Exact Expected (profit) = 2049.78

For this example, options 1 and 2 using variance A gives an estimated expected profit that is closest to the exact expected profit. Next, the expected profit values will be simulated in many replications to compare these values with the exact expected profit. Equation 20 can be used to compare the estimated expected profit to the exact expected profit. Table 7 presents the average different values from the exact solution.

$$APDE\ (E) = [Avg.\ |\ (E-Ev.\ E)|/\ (Ev.\ E)]*100 \qquad (20)$$

TABLE 7

Average Percent Difference from Exact Expected Profit

Structure 1		
Option	**Avg. Percent Difference from Exact Exp (profit)**	
	Method A	**Method B**
1	1.46	0.88
2	1.46	0.88
3	1.88	3.18
4	1.50	0.89
5	1.01	1.48

The same thing will be done for many different values of data input and yield for every option and process structure and then the results of all structures will be compared to find the best estimation method to use for predicting the output parameters in the real manufacturing process.

CHAPTER IV

RESULTS OF EXPERIMENTATION

This chapter presents the results of the experiment from eight structures that are summarized in the tables. In each structure, the table shows the average value of percentage different mean, variance, and expected profit from the methods A and B in 5 options. The result tables compare the options and show the appropriate method to estimate the output parameters (mean, variance, and expected profit).

Results

The eight process structures are shown below along with their results. Process structure 1 is shown in Figure 10. From the results shown in Table 8, the percentage of mean are equal in each method option except method B option 2 that has the highest value. The percentages of variance are very different especially method B option 3. When comparing between options, the method B option 4 is the closest to the actual value. For the expected profit percentage, method B options 1, and 2 are the best, but their percentages of variances are higher than method B option 4. Therefore the best option of structure 1 is method B option 4.

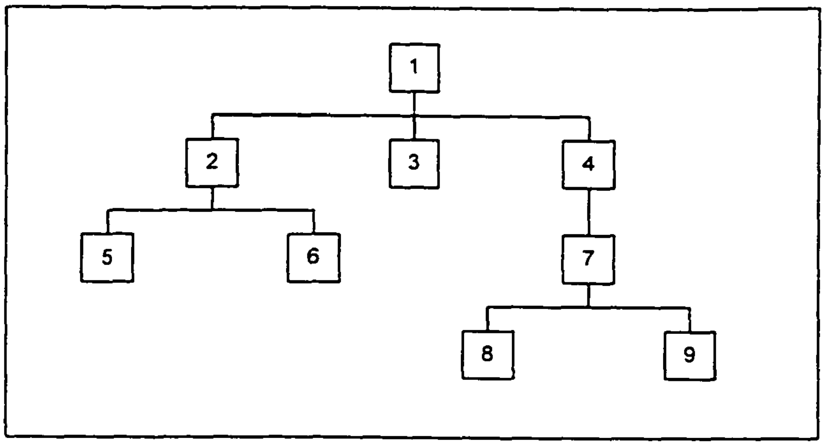

Figure 10 – Structure 1

TABLE 8

The Results of Structure 1

Structure 1				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**1.53**	21.14	1.46
	2	**1.53**	21.14	1.46
	3	**1.53**	68.31	1.88
	4	**1.53**	21.95	1.50
	5	**1.53**	17.84	1.01
B	1	**1.53**	10.18	**0.88**
	2	1.64	13.45	**0.88**
	3	**1.53**	119.46	3.18
	4	**1.53**	**9.21**	0.89
	5	**1.53**	37.43	1.48

Structure 2 (Figure 11) has one long serial process and one spread parallel process. It is not such a complex stage structure. It has small value of the mean and all of options are equal (see Table 9). For the variance values, they also are small except method B option 3, which has the large value of the different variance percentage.

For method B, options 1,2, and 4 provide good estimation. Obviously, method B option 4 has the minimum percentage of variance. Furthermore, this option gives the closest result of expected profit. It means method B option 4 is the most precise value of this structure.

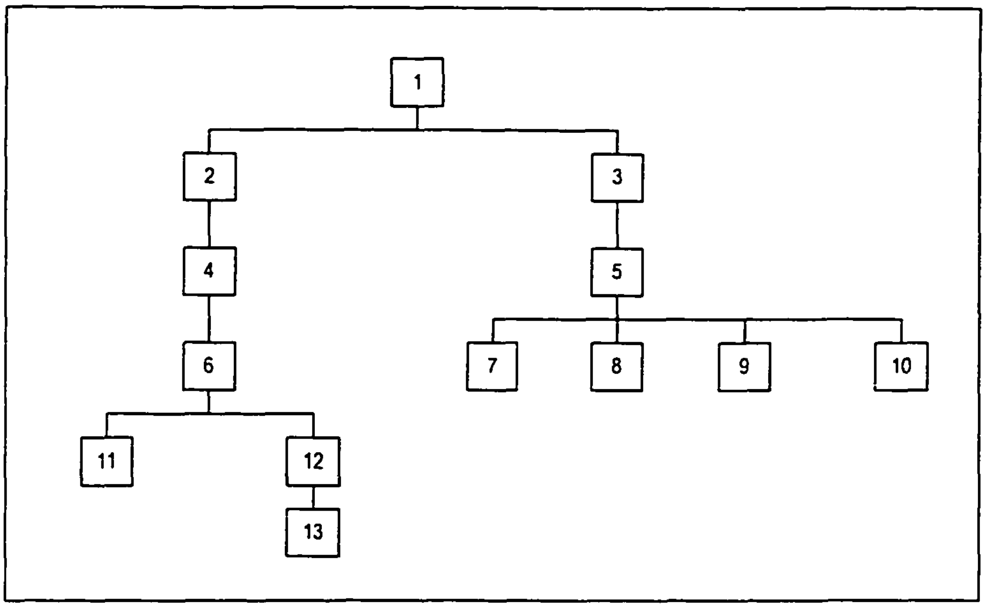

Figure 11 – Structure 2

TABLE 9

The Results of Structure 2

Structure 2				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**0.74**	23.05	4.86
	2	**0.74**	23.05	4.86
	3	**0.74**	49.90	10.44
	4	**0.74**	23.05	4.86
	5	**0.74**	14.83	2.58
B	1	**0.74**	7.77	3.03
	2	**0.74**	7.77	3.03
	3	**0.74**	89.22	17.10
	4	**0.74**	**7.07**	**2.03**
	5	**0.74**	42.43	7.34

Structure3 (Figure 12) is more complex than the structures that were described before. It provides large values of average percentage mean and large values of average variance percentage (see Table 10).

From the results, method B options 1,2, and 4 provide the closest value of variance and expected profit percentage to the actual value. They are the best estimation of this structure.

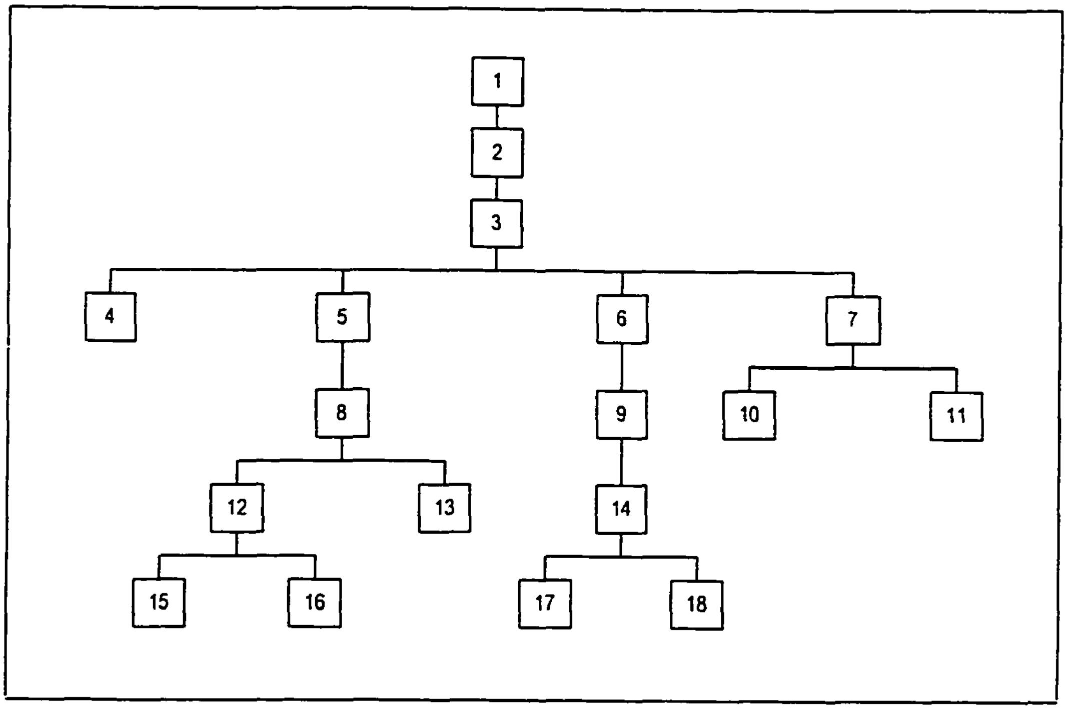

Figure 12 – Structure 3

TABLE 10

The Results of Structure 3

Structure 3				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**1.21**	47.12	12.66
	2	**1.21**	47.12	12.66
	3	**1.21**	58.37	13.30
	4	**1.21**	47.12	12.66
	5	**1.21**	11.07	5.57
B	1	**1.21**	**6.50**	**3.37**
	2	**1.21**	**6.50**	**3.37**
	3	**1.21**	146.43	27.58
	4	**1.21**	**6.50**	**3.37**
	5	**1.21**	80.36	16.34

Structure 4 (Figure 13) is the long vertical process. It seems like a form structure that has only two parallel and two serial structures in each step process. The result of mean percentage is very low compared to the options from other structures that have discussed (see Table 11). The variance percentage is very close to the actual value especially method B option 4. It can be concluded that the best estimation of this structure is method B option 4 because expected profit from the other options are not different.

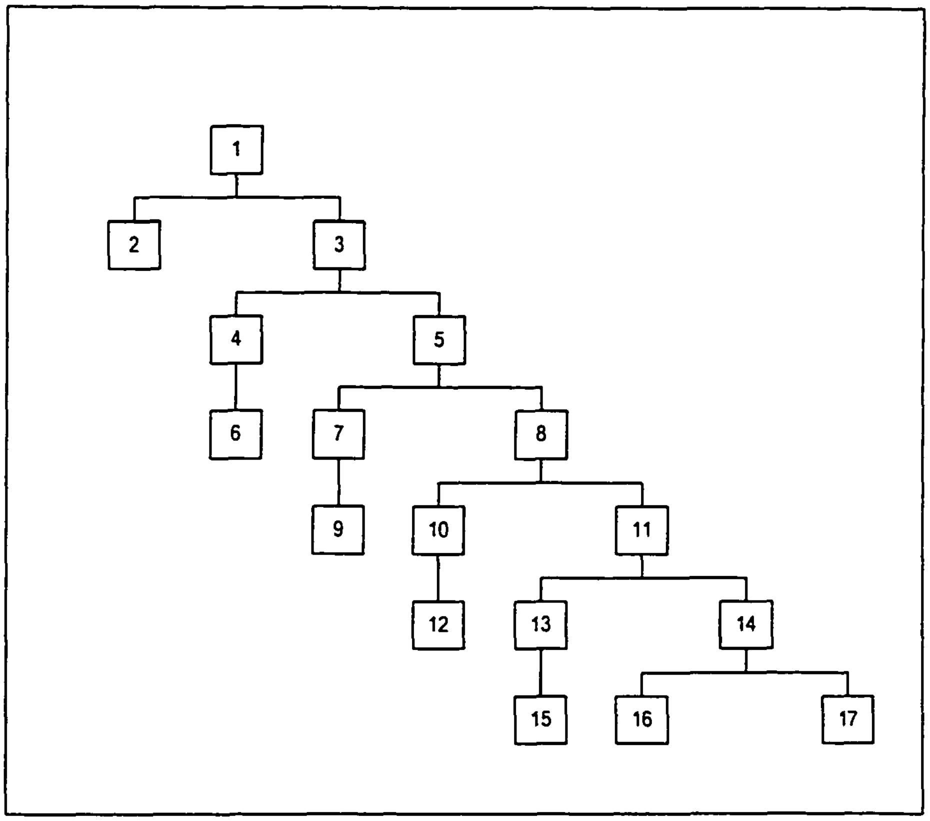

Figure 13 - Structure 4

TABLE 11

The Results of Structure 4

Structure 4				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**0.80**	45.58	28.76
	2	**0.80**	45.58	28.76
	3	**0.80**	47.56	46.91
	4	**0.80**	48.88	31.48
	5	**0.80**	6.84	10.99
B	1	**0.80**	6.61	**11.01**
	2	**0.80**	6.61	**11.01**
	3	**0.80**	82.76	70.36
	4	**0.80**	**2.27**	11.64
	5	**0.80**	43.41	48.69

Structure5 (Figure 14) has many parallel operations. Table 12 shows the values of the average percentage mean are equal and method A options 1 and 2 have the closest values of variance compared with the actual. For expected profit, the values in each option are not different. Thus, it can be concluded the method A options 1 and 2 are the best estimation for structure 5.

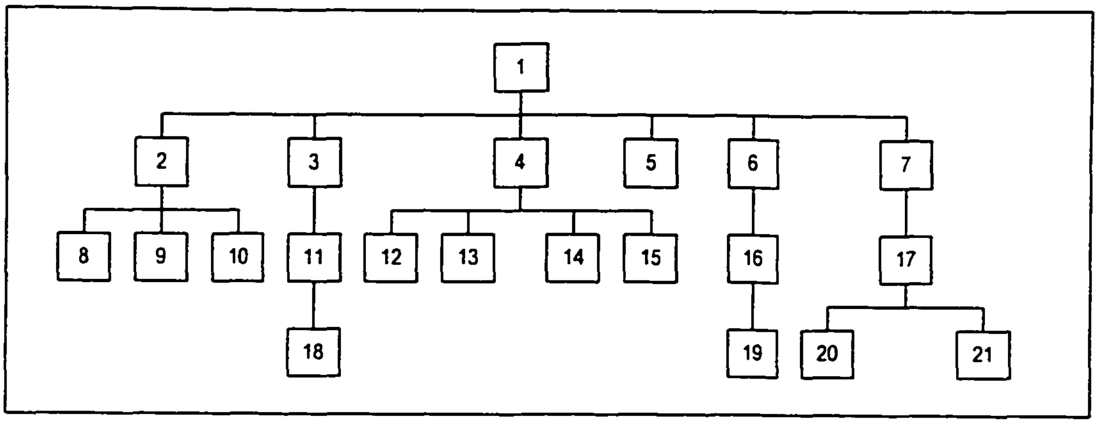

Figure 14 - Structure 5

TABLE 12

The Results of Structure 5

Structure 5				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**0.77**	**13.87**	1.91
	2	**0.77**	**13.87**	1.91
	3	**0.77**	99.74	2.46
	4	**0.77**	26.09	2.43
	5	**0.77**	36.30	1.87
B	1	**0.77**	14.86	1.86
	2	**0.77**	14.86	1.86
	3	**0.77**	129.33	2.43
	4	**0.77**	19.61	2.32
	5	**0.77**	59.67	**1.84**

Again, structure 6 (Figure 15) is another complex structure that has large value of average mean and average variance percentage (see Table 13). The method B option 4 is the best estimation of this structure because it has the smallest value of variance although the average expected profit is bigger than method B options 1and 2.

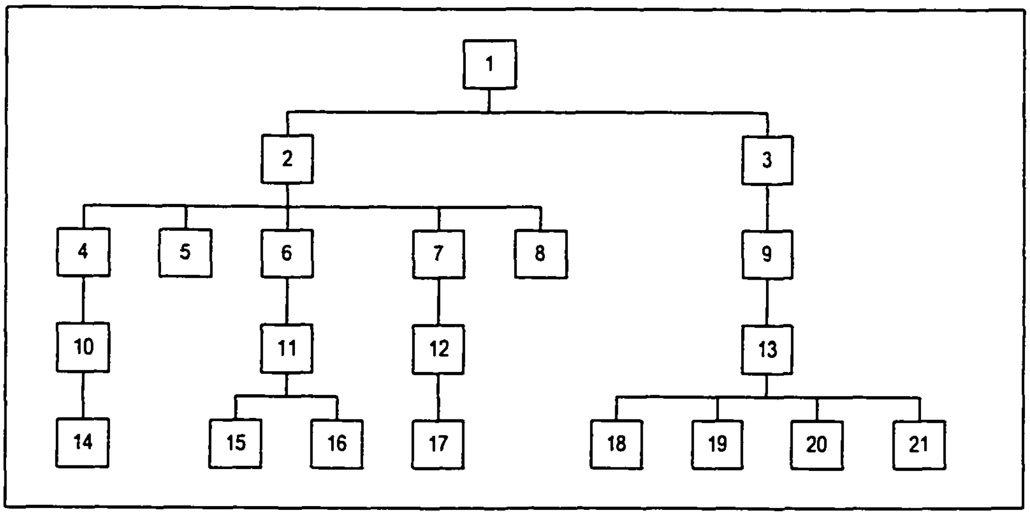

Figure 15 - Structure 6

TABLE 13

The Results of Structure 6

Structure 6				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**1.69**	27.11	7.84
	2	**1.69**	27.11	7.84
	3	**1.69**	125.79	11.44
	4	**1.69**	27.11	7.84
	5	**1.69**	26.57	6.68
B	1	**1.69**	13.95	**5.68**
	2	**1.69**	13.95	**5.68**
	3	**1.69**	178.08	13.53
	4	**1.69**	**12.29**	5.69
	5	**1.69**	75.55	8.39

Structure7 (Figure 17) is created to be a balance structure. The structure produces the lowest value of average mean percentage compared with other structures (see Table 14). Additionally, the average variance percentages are small for method B options 1,2, and 4.

For the expected profit percent, this structure has the closest value of all the structures especially method B options 1,2, and 4. Therefore, it can be concluded that the best estimation of this structure is method B options 1,2, and 4.

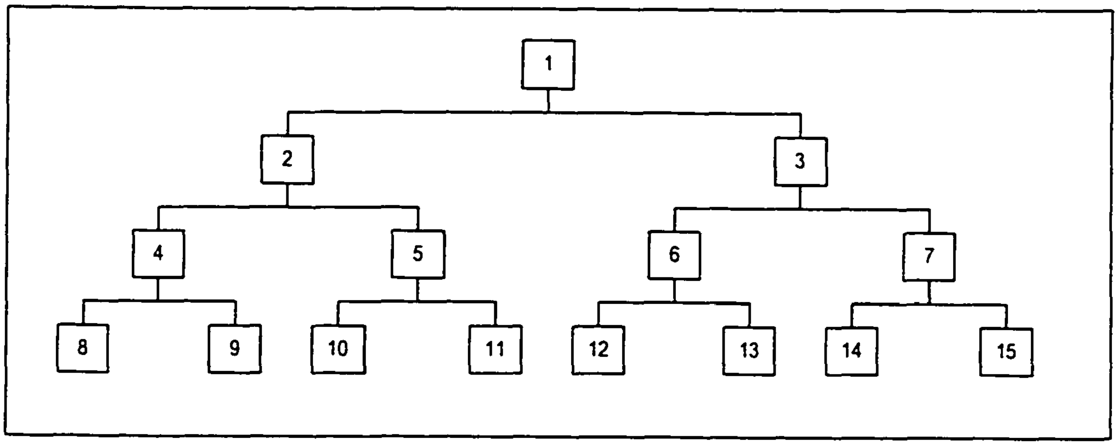

Figure 16 – Structure 7

TABLE 14

The Results of Structure 7

Structure 7				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**0.63**	24.24	5.00
	2	**0.63**	24.24	5.00
	3	**0.63**	66.07	8.32
	4	**0.63**	24.24	5.00
	5	**0.63**	21.22	4.87
B	1	**0.63**	**7.05**	**1.09**
	2	**0.63**	**7.05**	**1.09**
	3	**0.63**	104.91	12.97
	4	**0.63**	**7.05**	**1.09**
	5	**0.63**	58.17	8.75

For structure 8 (Figure 17), it is an unbalance structure because it has a serial process and the other side is a parallel process. This structure has high values of average mean percentage especially option 2 of methods A and B and high values of variance especially option 3 of methods A and B (see Table 15).

Method B option 4 is the best estimation method because it has the closest value of the average variance although it has a bigger value of the average expected profit than method B option 1.

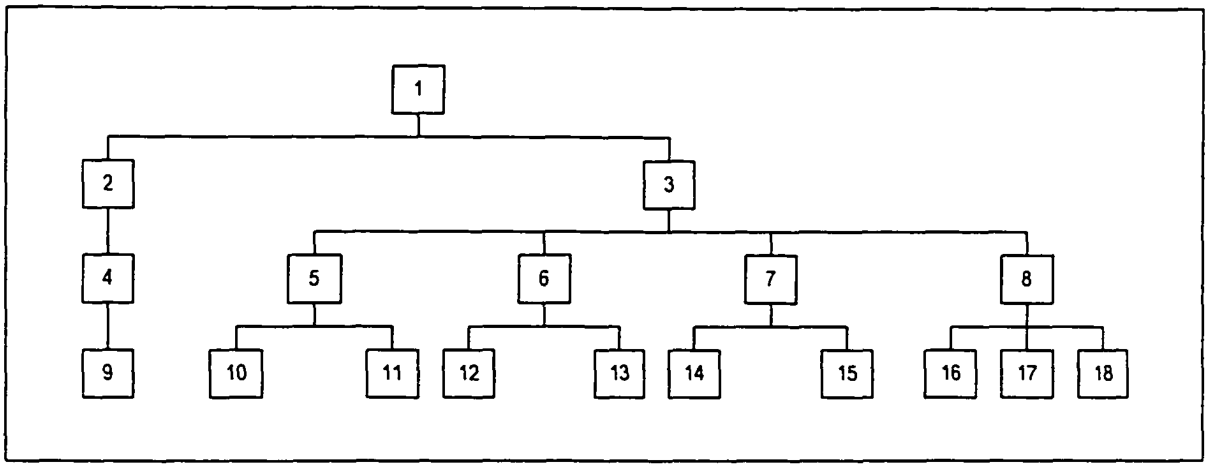

Figure 17- Structure 8

TABLE 15

The Results of Structure 8

Structure 8				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**1.35**	26.80	11.13
	2	5.86	23.78	21.69
	3	**1.35**	105.94	**4.69**
	4	**1.35**	26.80	11.13
	5	**1.35**	31.21	7.61
B	1	**1.35**	9.89	8.27
	2	5.86	13.94	19.47
	3	**1.35**	167.57	10.32
	4	**1.35**	**7.31**	8.36
	5	**1.35**	71.01	7.21

CHAPTER V

CONCLUSION AND FUTURE RESEARCH

This thesis develops a method to estimate the output parameters (mean and variance) and expected profit that is closest to the actual value for 8 multi-stage structures. To compare estimation method, the yields and the initial input are considered to simulate in different structures by using Monte Carlo Simulation.

The results are shown by the experiment in the previous chapter. The results can be summarized by evaluating the average percentage value of mean, variance, and expected profit. There are two ways of evaluation:

1) Evaluation from Parameters (mean, variance, and expected profit)
2) Evaluation from Structure

Evaluation from Parameters

The average percent difference from optimal values can be summarized across all 8 process structures. Figure 18 shows the average percent difference for the output mean for each estimation option and method.

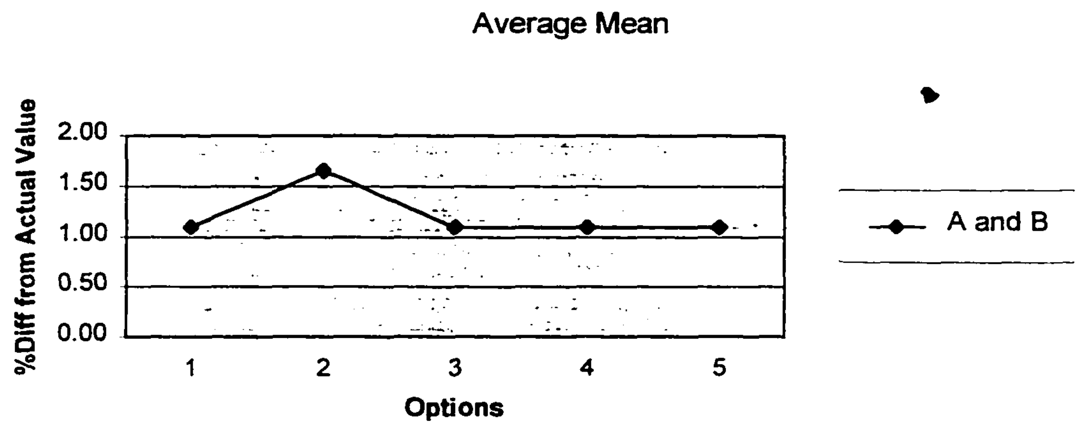

Figure 18 - The Average Mean

From the mean graph in Figure 18, methods A and B have the same result except option 2 because options 2 of structure 1 and 8 have a different mean percentage from the other options. Method A option 2 is a little better than method B option 2 about 0.01%, but not substantial enough improvement to determine which method is the best.

However, it may be concluded the method that uses the minimum value of mean in each line structure for this experiment is a good estimation of mean output because it has very small percentage of value only 1.09%. Furthermore, the average variance percentage of all structures should also be examined. See the variance graph in Figure 19.

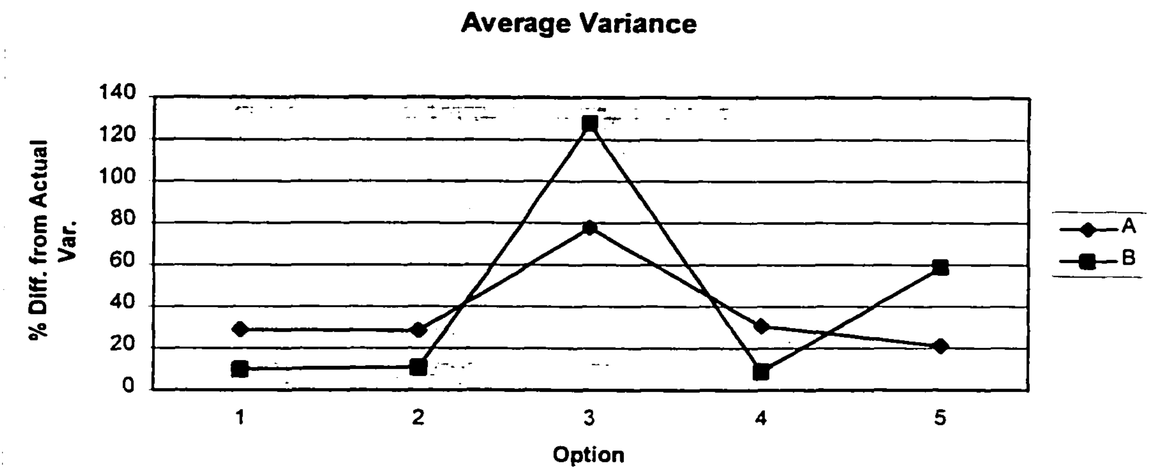

Figure 19 - The Average Variance

The graph shows method B option 4 is the best estimation method because it has the minimum value. It means that the minimum variance is the most appropriate method to estimate the variance output. Figure 20 shows the average percentage of expected profit.

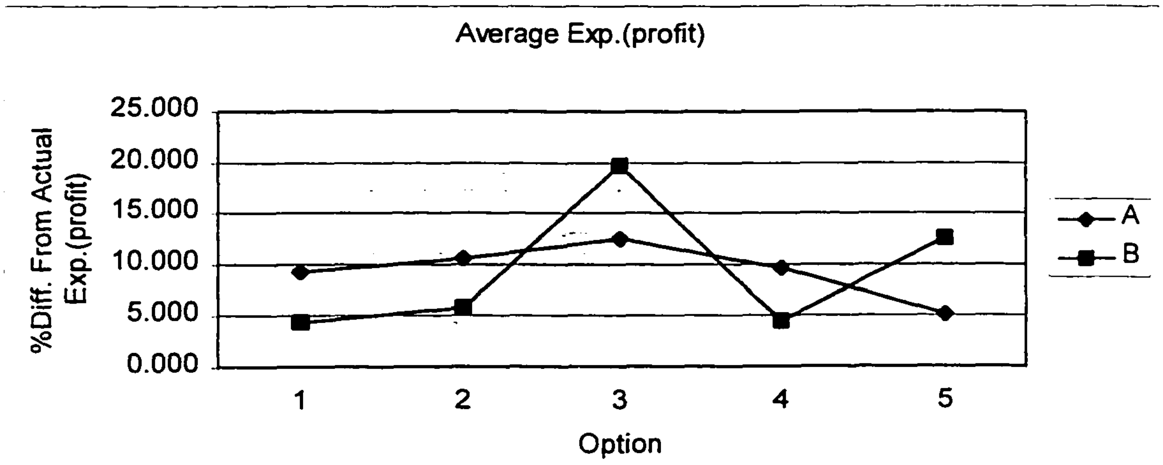

Figure 20 - The Average Exp.(profit)

From Figure 20, the expected profit graph looks like the variance graph. This graph shows the expected profit of method B option 1 and 4 are the best estimation method. Table 16 is the summary of all structures.

TABLE 16

The Summary of Structure 1-8

Summary Structure 1-8				
Method	**Option**	**Avg. Percent Different from Exact**		
		Mean	**Variance**	**Exp.(Profit)**
A	1	**1.09**	28.61	9.20
	2	1.65	28.24	10.52
	3	**1.09**	77.71	12.43
	4	**1.09**	30.66	9.61
	5	**1.09**	20.73	5.15
B	1	**1.09**	9.60	**4.40**
	2	1.67	10.52	5.80
	3	**1.09**	127.22	19.68
	4	**1.09**	**8.91**	4.42
	5	**1.09**	58.50	12.50

In conclusion, method B option 4 has the minimum value compared with the other options. Furthermore, these values will be appropriate to use in calculating the profit.

Evaluation from Structure

An evaluation score is computed by combining average percentage difference values of mean, variance, and expected profit for each structure. Therefore methods with a small evaluation scores more accurately estimate the actual output parameters than methods with large evaluation scores. To evaluate the estimation methods, equation (21) is created to present how accurate of method option in each structure close to the actual value, which is shown in Table 17.

M = average percentage difference from mean

V = average percentage difference from variance

E = average percentage difference from expected profit

$$\text{Evaluation Score} = M * V * E \tag{21}$$

TABLE 17

The Evaluation of Method Options

Evaluation Score									
Structure / **Method Option**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Sum**
A1	47.24	82.51	721.08	1043.39	**20.52**	359.18	75.94	402.66	2752.52
A2	47.24	82.51	721.08	1043.39	**20.52**	359.18	75.94	3023.93	5373.79
A3	196.93	383.41	938.48	1775.72	190.20	2432.38	344.66	670.87	6932.67
A4	50.46	82.51	721.08	1224.80	48.97	359.18	75.94	402.66	2965.59
A5	27.57	28.18	74.54	59.80	52.54	300.20	64.72	320.54	928.09
B1	13.67	17.34	**26.50**	57.94	21.37	133.89	**4.84**	110.50	386.05
B2	19.31	17.34	**26.50**	57.94	21.37	133.89	**4.84**	1590.70	1871.89
B3	580.81	1123.25	4882.58	4634.63	243.04	4074.24	852.86	2334.45	18725.86
B4	**12.48**	**10.58**	**26.50**	**21.03**	35.23	**118.19**	**4.84**	**82.42**	**311.26**
B5	84.56	229.38	1587.64	1682.09	85.04	1071.31	319.19	690.72	5749.93

From Table 17, the results can be summarized as follows.

- For structure 1, the best estimation is method B option 4.
- For structure 2, the best estimation is method B option 4.
- For structure 3, the best estimation is method B options 1,2, and 4.
- For structure 4, the best estimation is method B option 4.
- For structure 5, the best estimation is method A options 1 and 2.
- For structure 6, the best estimation is method B option 4.
- For structure 7, the best estimation is method B options 1, 2, and 4.
- For structure 8, the best estimation is method B option 4.
- For the summary, the best estimation is method B option 4.

From both of evaluations, method B option 4 appears to be the best using minimum mean and minimum variance of output parameters to calculate the expected profit is the best alternative for estimating the parameters for all 8 structures. Although, the method B option 4 is the best, it does not give a good prediction for the structure 5 because structure 5 has wide parallel processes. Furthermore the prediction should be concerned the more complex the structure, the more possible estimation error.

Future research

From the conclusion of the experiment, the mean and variance output parameters nor the expected cannot be exactly estimated. The goal of the method developed in this thesis is to estimate the value quickly in various systems.

The method that is the best for this experiment is method B option 4, which uses the minimum mean and variance. This method can calculate the output parameters faster, but it cannot be proved this method is the most accurate or the fastest method. New methods are still being developed.

An alternative method of estimating the output parameters of a process structure should be investigated in the future. This alternative method uses a different approach to traverse the structure and calculate of the mean and variance. This alternative method is demonstrated in the following example

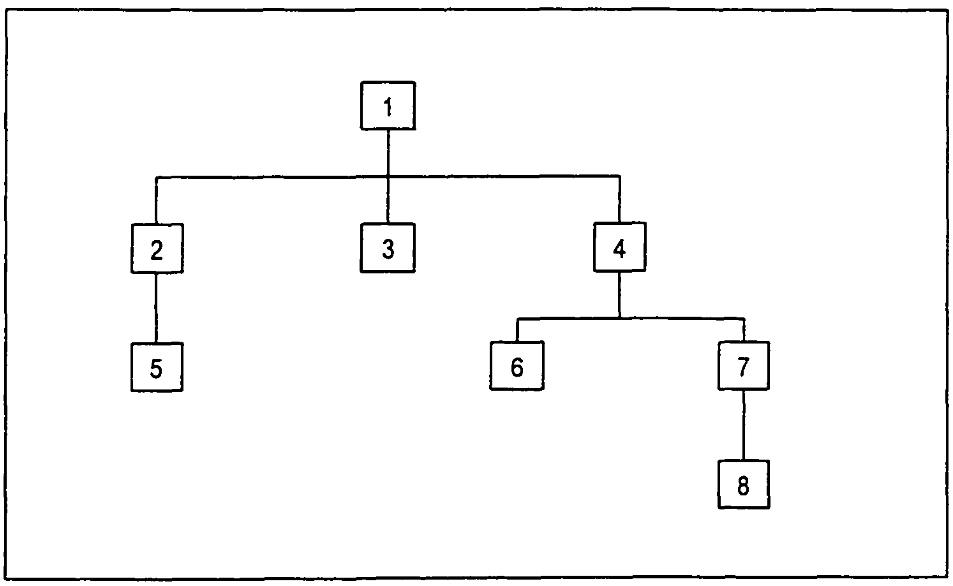

Figure 21 – Example Structure

The Methodology

1) The methodology will start at the bottom of the structure and go step by step up to the top of the structure.
2) In step 1, it uses serial formulas to find mean and variance parameters of stage 7 and 8.
3) In step 2, it uses parallel formulas to find mean and variance parameters of stage 6 and the result of step 1 (output of stage 7 and 8)
4) In step 3, it uses series formulas to find the mean and variance parameters of stage 2 and 5.
5) In step 4, it uses series formulas to find mean and variance parameters of stage 4 and step 2 (output of stage 6,7, and 8).
6) In step 5, it uses parallel formulas to find mean and variance parameters of stage 3 and step 3 (output of stage 2 and 5).
7) In the final step, it uses series formulas to find mean and variance parameters of stage 1 and step 5 (output of stage 2,3,4,5,6,7,and 8).

This alternative structure evaluation method should be investigated and the estimated output parameters compared to the exact output parameters.

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APPENDIX A

STRUCTURE 1

	Input	L1	L2	L3	L4	Mean	Var.
Path 1	123	1	0.75	0.96	0.88	77.93	25.25
Path 2	85	1	0.65	0.96	0.88	46.68	18.36
Path 3	130	1	1	0.93	0.88	106.39	18.43
Path 4	211	0.67	0.78	0.91	0.88	88.30	37.32
Path 5	96	0.78	0.78	0.91	0.88	46.77	18.22

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	46.68	18.36	40	100	30	2	30	46.68	4.29	1773.15	-1.56	1.59
Opt2	46.68	18.36	40	100	30	2	30	46.68	4.29	1773.15	-1.56	1.59
Opt3	46.68	37.32	40	100	30	2	30	46.68	6.11	1773.15	-1.09	1.17
Opt4	46.68	18.22	40	100	30	2	30	46.68	4.27	1773.15	-1.56	1.59
Opt5	46.68	23.52	40	100	30	2	30	46.68	4.85	1773.15	-1.38	1.42

E 1(Profit)	2007.53
E 2(Profit)	2007.53
E 3(Profit)	1952.4
E 4(Profit)	2005.5
E 5(Profit)	1994.19

STRUCTURE 2

	Input	L1	L2	L3	L4	L5	L6	Mean	Var.
Path 1	247	1	0.77	0.98	0.73	0.83	0.79	89.22	37.37
Path 2	199	0.85	0.96	0.98	0.73	0.83	0.79	76.17	31.46
Path 3	163	1	1	0.7	0.86	0.78	0.79	60.47	25.55
Path 4	137	1	1	0.82	0.86	0.78	0.79	59.53	24.06
Path 5	127	1	1	0.99	0.86	0.78	0.79	66.63	25.19
Path 6	120	1	1	0.91	0.86	0.78	0.79	57.87	22.59

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	57.87	22.59	50	80	30	5	30	57.87	4.75	9381.02	-1.66	1.68
Opt2	57.87	22.59	50	80	30	5	30	57.87	4.75	9381.02	-1.66	1.68
Opt3	57.87	37.37	50	80	30	5	30	57.87	6.11	9381.02	-1.29	1.34
Opt4	57.87	22.59	50	80	30	5	30	57.87	4.75	9381.02	-1.66	1.68
Opt5	57.87	27.71	50	80	30	5	30	57.87	5.26	9381.02	-1.49	1.53

E 1(Profit)	-5633.47
E 2(Profit)	-5633.47
E 3(Profit)	-5659.27
E 4(Profit)	-5633.47
E 5(Profit)	-5642.42

APPENDIX B

STRUCTURE 3

	Input	L1	L2	L3	L4	L5	L6	L7	Mean	Var.
Path 1	144	1	1	1	0.92	0.78	0.74	0.86	65.76	26.52
Path 2	157	0.83	0.88	0.78	0.75	0.78	0.74	0.86	33.30	12.38
Path 3	149	0.89	0.88	0.78	0.75	0.78	0.74	0.86	33.89	12.88
Path 4	122	1	0.79	0.78	0.75	0.78	0.74	0.86	27.99	10.77
Path 5	248	0.81	0.83	0.98	0.79	0.78	0.74	0.86	64.08	25.19
Path 6	105	0.72	0.83	0.98	0.79	0.78	0.74	0.86	24.11	9.28
Path 7	152	1	1	0.96	0.83	0.78	0.74	0.86	60.12	24.79
Path 8	174	1	1	0.71	0.83	0.78	0.74	0.86	50.90	20.99

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	24.11	9.28	45	150	85	1	77	24.11	3.05	1990.76	6.86	0.00
Opt2	24.11	9.28	45	150	85	1	77	24.11	3.05	1990.76	6.86	0.00
Opt3	24.11	26.52	45	150	85	1	77	24.11	5.15	1990.76	4.06	0.00
Opt4	24.11	9.28	45	150	85	1	77	24.11	3.05	1990.76	6.86	0.00
Opt5	24.11	17.85	45	150	85	1	77	24.11	4.23	1990.76	4.94	0.00

E 1(Profit)	-149.24
E 2(Profit)	-149.24
E 3(Profit)	-149.50
E 4(Profit)	-149.24
E 5(Profit)	-149.38

STRUCTURE 4

	Input	L1	L2	L3	L4	L5	L6	L7	Mean	Var.
Path 1	137	1	1	1	1	1	0.71	0.84	81.71	29.19
Path 2	137	1	1	1	0.83	0.89	0.79	0.84	67.16	28.13
Path 3	155	1	1	0.85	0.7	0.99	0.79	0.84	60.59	25.50
Path 4	244	1	0.69	0.65	0.86	0.99	0.79	0.84	61.83	25.42
Path 5	144	0.66	0.86	0.99	0.86	0.99	0.79	0.84	45.72	19.20
Path 6	175	0.97	0.84	0.99	0.86	0.99	0.79	0.84	79.76	31.48
Path 7	191	0.84	0.84	0.99	0.86	0.99	0.79	0.84	75.38	30.51

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	45.72	19.20	50	120	30	2	30	45.72	4.38	4415.72	0.98	0.09
Opt2	45.72	19.20	50	120	30	2	30	45.72	4.38	4415.72	0.98	0.09
Opt3	45.72	31.48	50	120	30	2	30	45.72	5.61	4415.72	0.76	0.13
Opt4	45.72	19.20	50	120	30	2	30	45.72	4.38	4415.72	0.98	0.09
Opt5	45.72	27.06	50	120	30	2	30	45.72	5.20	4415.72	0.82	0.12

E 1(Profit)	872.31
E 2(Profit)	872.31
E 3(Profit)	811.69
E 4(Profit)	872.31
E 5(Profit)	833.25

APPENDIX C

STRUCTURE 5

	Input	L1	L2	L3	L4	Mean	Var.
Path 1	116	1	0.78	0.81	0.96	70.36	23.82
Path 2	174	1	0.69	0.81	0.96	93.36	36.34
Path 3	132	1	0.87	0.81	0.96	89.30	25.72
Path 4	205	0.73	0.8	0.7	0.96	80.45	34.42
Path 5	149	1	0.93	0.68	0.96	90.46	32.27
Path 6	203	1	0.89	0.68	0.96	117.94	43.57
Path 7	237	1	0.89	0.68	0.96	137.70	50.87
Path 8	215	1	0.88	0.68	0.96	123.51	46.02
Path 9	229	1	1	0.91	0.96	200.05	24.57
Path 10	206	0.72	0.88	0.84	0.96	105.25	40.73
Path 11	238	0.85	0.95	0.75	0.96	138.37	48.60
Path 12	166	0.76	0.95	0.75	0.96	86.29	33.35

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	70.36	23.82	80	120	56	2	50	70.36	4.88	6733.12	1.98	0.01
Opt2	70.36	23.82	80	120	56	2	50	70.36	4.88	6733.12	1.98	0.01
Opt3	70.36	50.87	80	120	56	2	50	70.36	7.13	6733.12	1.35	0.04
Opt4	70.36	23.82	80	120	56	2	50	70.36	4.88	6733.12	1.98	0.01
Opt5	70.36	36.69	80	120	56	2	50	70.36	6.06	6733.12	1.59	0.02

E 1(Profit)	1159.6
E 2(Profit)	1159.6
E 3(Profit)	1103.83
E 4(Profit)	1159.6
E 5(Profit)	1137.17

STRUCTURE 6

	Input	L1	L2	L3	L4	L5	Mean	Var.
Path 1	116	0.86	0.81	0.93	0.97	0.67	48.84	20.48
Path 2	243	1	1	0.83	0.97	0.67	131.08	51.49
Path 3	88	0.87	0.77	0.96	0.97	0.67	36.78	15.58
Path 4	88	0.85	0.77	0.96	0.97	0.67	35.93	15.27
Path 5	93	0.91	0.88	0.85	0.97	0.67	41.14	17.02
Path 6	119	1	1	0.91	0.97	0.67	70.38	25.90
Path 7	174	0.85	0.87	0.73	0.84	0.67	52.86	22.14
Path 8	138	0.86	0.87	0.73	0.84	0.67	42.42	17.80
Path 9	241	0.85	0.87	0.73	0.84	0.67	73.22	30.67
Path 10	93	0.82	0.87	0.73	0.84	0.67	27.26	11.36

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	27.26	11.36	30	120	56	2	50	27.26	3.37	4304.16	0.81	0.12
Opt2	27.26	11.36	30	120	56	2	50	27.26	3.37	4304.16	0.81	0.12
Opt3	27.26	51.49	30	120	56	2	50	27.26	7.18	4304.16	0.38	0.23
Opt4	27.26	11.36	30	120	56	2	50	27.26	3.37	4304.16	0.81	0.12
Opt5	27.26	22.77	30	120	56	2	50	27.26	4.77	4304.16	0.57	0.18

E 1(Profit)	-1276.72
E 2(Profit)	-1276.72
E 3(Profit)	-1562.04
E 4(Profit)	-1276.72
E 5(Profit)	-1377.76

APPENDIX D

STRUCTURE 7

	Input	L1	L2	L3	L4	Mean	Var.
Path 1	237	0.87	0.85	0.87	0.94	143.33	46.73
Path 2	215	0.8	0.85	0.87	0.94	119.56	42.53
Path 3	229	0.93	0.82	0.87	0.94	142.82	45.25
Path 4	206	0.89	0.82	0.87	0.94	122.95	40.83
Path 5	238	0.89	0.81	0.7	0.94	112.90	45.55
Path 6	166	0.88	0.81	0.7	0.94	77.86	31.53
Path 7	177	0.95	0.77	0.7	0.94	85.19	34.60
Path 8	126	0.81	0.77	0.7	0.94	51.71	21.77

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	51.71	21.77	50	120	90	2	85	51.71	4.67	4954.55	-0.37	0.61
Opt2	51.71	21.77	50	120	90	2	85	51.71	4.67	4954.55	-0.37	0.61
Opt3	51.71	46.73	50	120	90	2	85	51.71	6.84	4954.55	-0.25	0.54
Opt4	51.71	21.77	50	120	90	2	85	51.71	4.67	4954.55	-0.37	0.61
Opt5	51.71	38.60	50	120	90	2	85	51.71	6.21	4954.55	-0.28	0.55

E 1(Profit)	563.71
E 2(Profit)	563.71
E 3(Profit)	310.64
E 4(Profit)	563.71
E 5(Profit)	388.22

STRUCTURE 8

	Input	L1	L2	L3	L4	Mean	Var.
Path 1	116	0.86	0.88	0.85	0.73	54.47	21.63
Path 2	243	0.87	0.87	0.91	0.73	122.18	47.16
Path 3	88	0.85	0.87	0.91	0.73	43.23	16.88
Path 4	88	0.91	0.93	0.91	0.73	49.47	17.88
Path 5	93	0.85	0.93	0.91	0.73	48.84	18.48
Path 6	119	0.86	0.83	0.91	0.73	56.43	22.38
Path 7	174	0.85	0.83	0.91	0.73	81.55	32.49
Path 8	138	0.82	0.96	0.91	0.73	72.17	27.55
Path 9	241	0.81	0.96	0.91	0.73	124.49	47.91
Path 10	93	0.77	0.96	0.91	0.73	45.67	18.14

	Mean	Var.	T	G	S	C	V	mean	σ	x bar	z	L(z)
Opt1	43.23	16.88	50	120	90	2	85	43.23	4.11	4065.29	1.65	0.02
Opt2	43.23	16.88	50	120	90	2	85	43.23	4.11	4065.29	1.65	0.02
Opt3	43.23	47.91	50	120	90	2	85	43.23	6.92	4065.29	0.98	0.09
Opt4	43.23	16.88	50	120	90	2	85	43.23	4.11	4065.29	1.65	0.02
Opt5	43.23	27.05	50	120	90	2	85	43.23	5.20	4065.29	1.30	0.05

E 1(Profit)	487.44
E 2(Profit)	487.44
E 3(Profit)	332.92
E 4(Profit)	487.44
E 5(Profit)	443.20

VITA

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